Gravitational waves production during the Big Bounce

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29 April, 2008

Motivation

Hunting for quantum gravitational effects.

For now there is no experimental evidence of quantum gravity.
Theory without experiment is pure mathematics.

Can it change in the near future?

LHC reach 14 TeV, we need 10^{16} TeV. We newer reach it with the present generation of accelerators.

We must to see high to see what is deep.

We need precise cosmological and astrophysical observations: Cosmic Microwave Background, Large Scale Structures, Gamma Bursts.

Gravitational waves produced during quantum epoch can survive till now living frozen on the super-horizontal scales!

Flat FRW metric with tensor perturbations take a form

$$ds^2 = a(\tau)^2(\eta_{\mu\nu} + h_{\mu\nu})dx^{\mu}dx^{\nu}.$$

where $|h_{\mu\nu}| \ll 1$. Based on the above metric and the Einstein equations we derive equation of motion

$$h''_{\mu\nu} + 2\mathcal{H}h'_{\mu\nu} + k^2 h_{\mu\nu} = 0$$

where

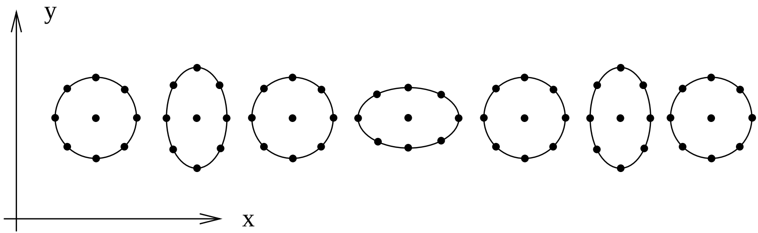
$$\mathcal{H} = \frac{a'}{a}.$$

$$h_{\mu\nu} = h_{\oplus} e^1_{\mu\nu} + h_{\otimes} e^2_{\mu\nu}$$

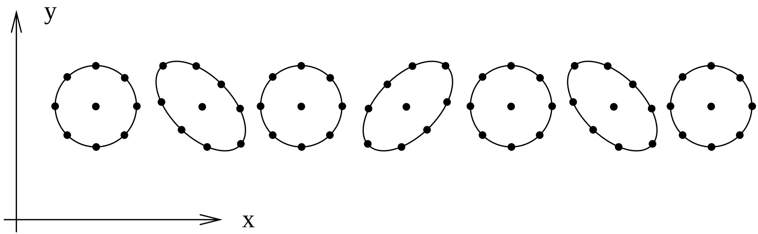
$$e^1_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$e^2_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Polarization $\oplus$:



Polarization $\otimes$:



Introducing new variable

$$u = \frac{ah_{\oplus}}{\sqrt{16\pi G}} = \frac{ah_{\otimes}}{\sqrt{16\pi G}}$$

we rewrite equation

$$h_i^{''} + 2\mathcal{H}h_i^{'} + k^2 h_i = 0$$

where $i = \oplus, \otimes$, to the form

$$u^{''} + \left[k^2 - \frac{a^{''}}{a}\right] u = 0.$$

- $k^2 \gg |a^{''}/a|$

  Solutions are decaying plane waves

$$h_i \simeq \frac{e^{\pm ik\tau}}{a}.$$

- $k^2 \ll |a^{''}/a|$

  Solutions describe what is often named super-horizontal amplifications

$$h_i \simeq A_k + B_k \int^{\tau} \frac{dx}{a^2(x)}.$$

Quantum fluctuations

Filed u can be treated as the quantum field initially in the vacuum state $|0\rangle$.

Vacuum state $|0\rangle$ = state without real particles.
However in the cosmological background no preferred vacuum state.

How to create particles from quantum vacuum?
We need horizon to disconnect initially causally connected quantum fluctuations.

- Hawking radiation. $T_H = \frac{\hbar c^3}{8\pi GMk}$
- Unruh effect. $T_U = \frac{\hbar \ddot{x}}{2\pi ck}$
- Cosmological particles creation. $T \sim H^{-1}$
  Quantum fluctuations red-shifted to the super horizontal scales becomes classical particles. This process lead to the structures formation in the Universe. Good observational evidence !

For the modes $\lambda \ll R_H$ we have simple harmonic oscillator

$$u'' + k^2 u = 0.$$

We can quantize this system introducing

$$\hat{u} = f(k,\tau)\hat{a} + f^*(k,\tau)\hat{a}^\dagger$$

where $f(k,\tau)$ satisfies classical equation of motion and the creation and annihilation operators satisfy

$$[\hat{a}, \hat{a}^\dagger] = 1, \quad \hat{a}|0\rangle = 0.$$

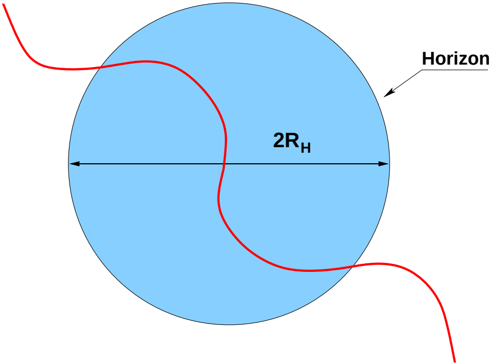
With use of the commutator $[\hat{u}, \hat{\pi}] = i$ where $\hat{\pi} = \hat{u}'$ we obtain

$$f(k,\tau) = \frac{1}{\sqrt{2k}} e^{-ik\tau}.$$

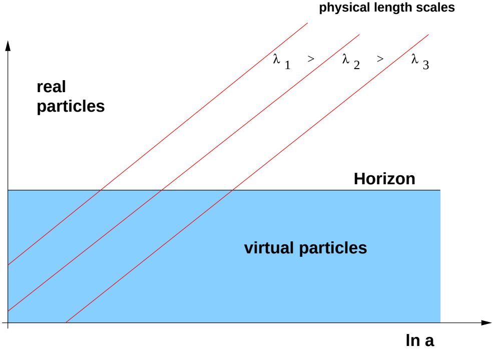
Zero point fluctuations of ground state

$$\begin{aligned} \langle u^2 \rangle &= \langle 0|\hat{u}^\dagger \hat{u}|0\rangle \\ &= \langle 0|\hat{a}^\dagger \hat{a}|0\rangle |f(k,\tau)|^2 \\ &= |f(k,\tau)|^2 = \frac{1}{2k}. \end{aligned}$$

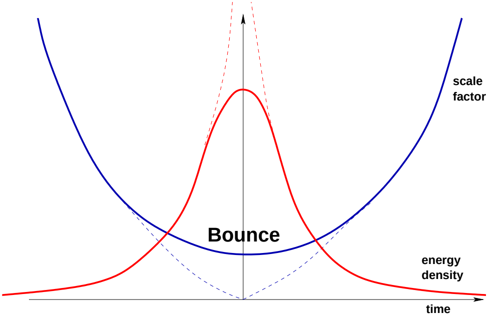
An example of a mode whose wavelength λ is comparable to the horizon radius, $\lambda \sim 2R_H$.



During the Inflation phase horizon scales R_H are nearly constant.
This produce nearly flat spectrum of perturbations.
In great agreement with CMB observations. However for low energy modes some deviations appear.



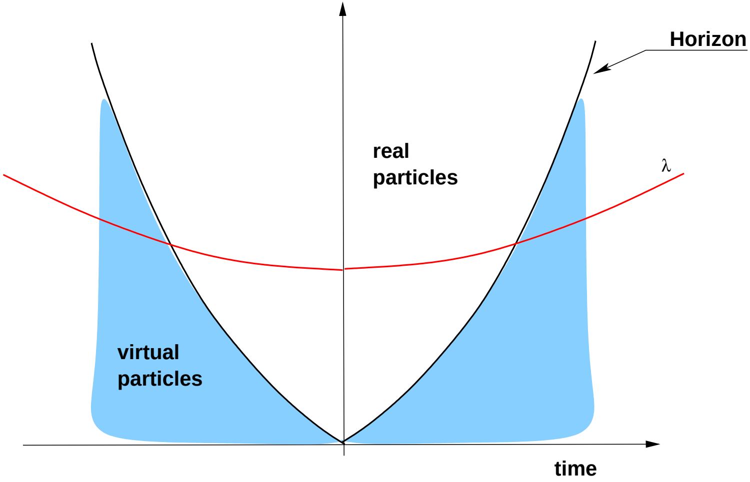
Loop Quantum Cosmology



Effective Friedmann equation:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right)$$

Horizon in the bouncing universe



- We perturb Ashtekar variables

$$
\begin{aligned}
E &= \bar{E} + \delta E \\
A &= \bar{A} + \delta A
\end{aligned}
$$

and extract part for the tensor modes.
- We add quantum holonomy corrections.
- Finlay we obtain

$$h_i'' + 2\mathcal{H}h_i' + k^2 h_i + T_Q h_i = 0$$

where

$$T_Q = -2\left(\frac{\bar{p}}{\bar{\mu}}\frac{\partial\bar{\mu}}{\partial\bar{p}}\right)\bar{\mu}^2\gamma^2\left[\frac{\sin\left(\bar{\mu}\gamma\bar{k}\right)}{\bar{\mu}\gamma}\right]^4 .$$

Correlation function:

$$\langle 0|\hat{h}^a_b(\vec{x},\tau)\hat{h}^b_a(\vec{y},\tau)|0\rangle = 4\frac{16\pi G}{a^2}\int\frac{d^3k}{(2\pi)^3}|f(k,\tau)|^2 e^{-i\vec{k}\cdot\vec{r}}$$
$$= \int\frac{dk}{k}\mathcal{P}_T(k,\tau)\frac{\sin kr}{kr}$$

where power spectrum

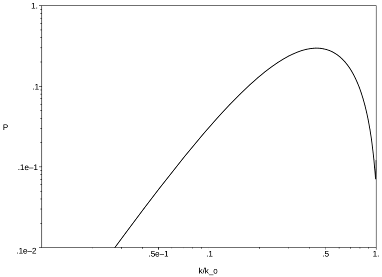
$$\mathcal{P}_T(k,\tau) = \frac{64\pi G}{a^2}\frac{k^3}{2\pi^2}|f(k,\tau)|^2.$$

We calculate spectrum on the horizon crossing, so $\tau(k)$. For $k \to 0$

$$\mathcal{P}_T(k) = \sqrt{\frac{12}{\pi}}|H_0^{(2)}(2\pi)|^2\left(\frac{k}{k_*}\right)^3$$

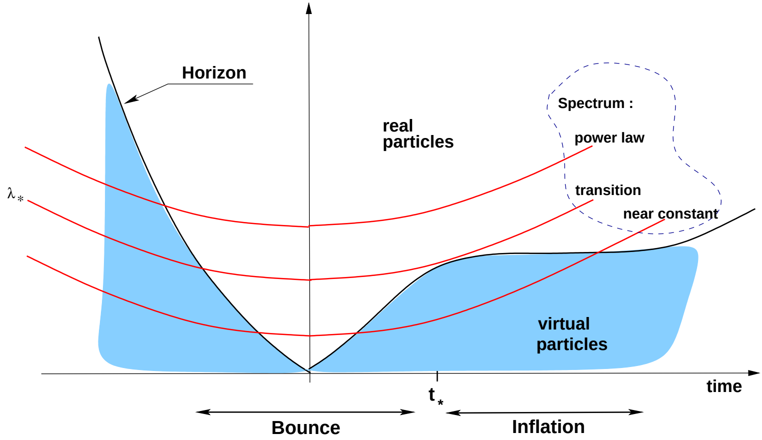
We have found analytical model of particles creation during the bounce for some specific setup. For this model expression for the power spectrum take a form

$$\mathcal{P}_T(k) = \frac{16Gk_0^2}{\pi}\left(\frac{k}{k_0}\right)^3 \frac{\mathrm{ch}\left[4\pi\sqrt{(k_0/k)^2-1}\right]}{\sqrt{1-(k/k_0)^2}\,\mathrm{ch}^2\left(\frac{2\pi}{k/k_0}\right)}.$$



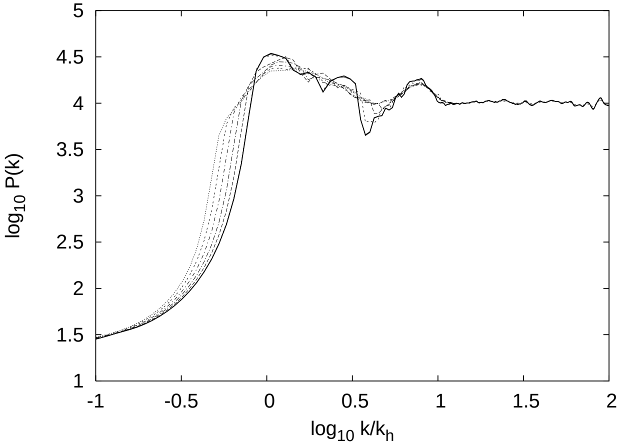
Bounce+Inflation

This model predict damping in the low energy part of the spectrum.



Largest scale structures has origin in the pre-Big Bang phase!

Observational spectrum of primordial perturbations [1]

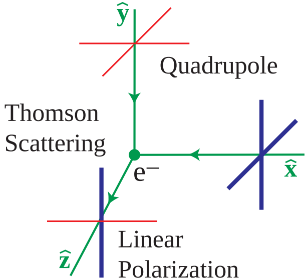


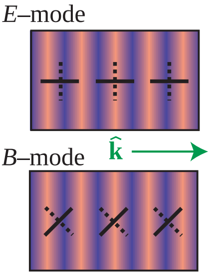
[1] Picture adopted from: Phys. Rev. D **70** (2004) 043523 [arXiv:astro-ph/0312174]

CMB polarisation

Thompson scattering

$$\frac{d\sigma}{d\Omega} = \frac{3\sigma_T}{8\pi}|\hat{e}' \cdot \hat{e}|^2$$





E = scalar+tensor, B = tensor.

B polarization is expected to observe by Planck satellite!

Summary and Outlook

- Quantum gravity effects can be potentially observed.
- We have found analytical model of gravitons creation during bounce.
- For more realistic models we need more numerical studies.
- LIGO observatory rather non adequate for the relic gravitational waves.
- Big bounce phase lead to the low multipoles damping observed in the CMB.
- Next step - polarization spectra.
- We are waiting for the Planck mission launch (September 2008). First results expected in three years.

Bibliography

- J. Mielczarek and M. Szydlowski,
  "Relic gravitons as the observable for Loop Quantum Cosmology,"
  Phys. Lett. B **657** (2007) 20 [arXiv:0705.4449 [gr-qc]].
- M. Bojowald and G. M. Hossain,
  "Loop quantum gravity corrections to gravitational wave dispersion,"
  Phys. Rev. D **77** (2008) 023508 [arXiv:0709.2365 [gr-qc]].
- J. Mielczarek and M. Szydlowski,
  "Relic gravitons from super-inflation,"
  arXiv:0710.2742 [gr-qc].