$\mathcal{O}(p^4)$ correction terms to the masses of the lightest hadrons from Chiral Perturbation Theory

Jakub Mielczarek

Marian Smoluchowski Institute of Physics, Jagiellonian University

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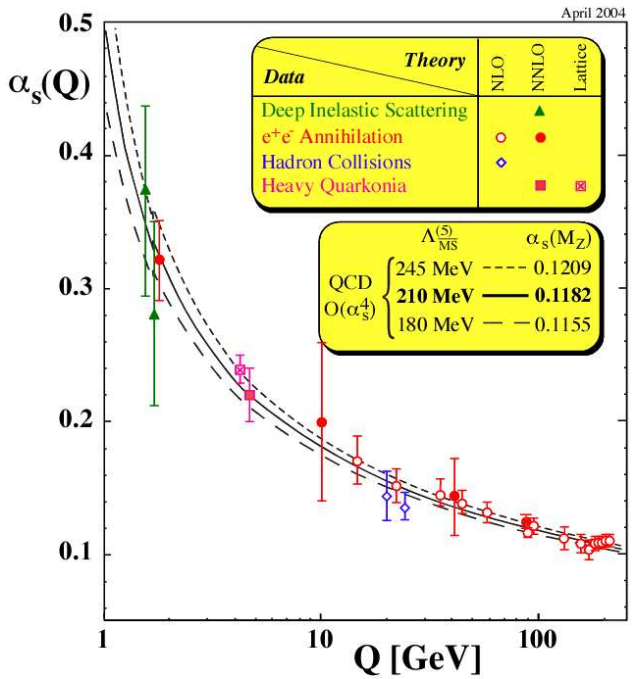
Effective Field Theories

dynamics at low energies (large distances) does not depend on the details of the dynamics at high energies (short distances)

$$\mathcal{L}_{QCD} \rightarrow \mathcal{L}_{\chi PT}$$

quarks, gluons $\rightarrow$ mesons, baryons

running coupling constant for QCD



$$\mathcal{L}_{QCD} = \sum_f \bar{q}_f (i\not{D} - m) q_f - \frac{1}{4} G_{\mu\nu,\alpha} G^{\mu\nu}_{\alpha}$$

QCD vacuum state spontaneously breaks the chiral symmetry

$$SU(3)_L \otimes SU(3)_R \longrightarrow SU(3)_V$$

$$SU(3)_A \Downarrow$$

Meson octet

$$\mathbf{\Phi} = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$

Hadron masses can be calculated with $\mathcal{L}_{EFT}$

$$iS(p) = \frac{i}{\not{p} - m_0 - \Sigma(p) + i\epsilon}$$

Σ dependence on the quark masses!

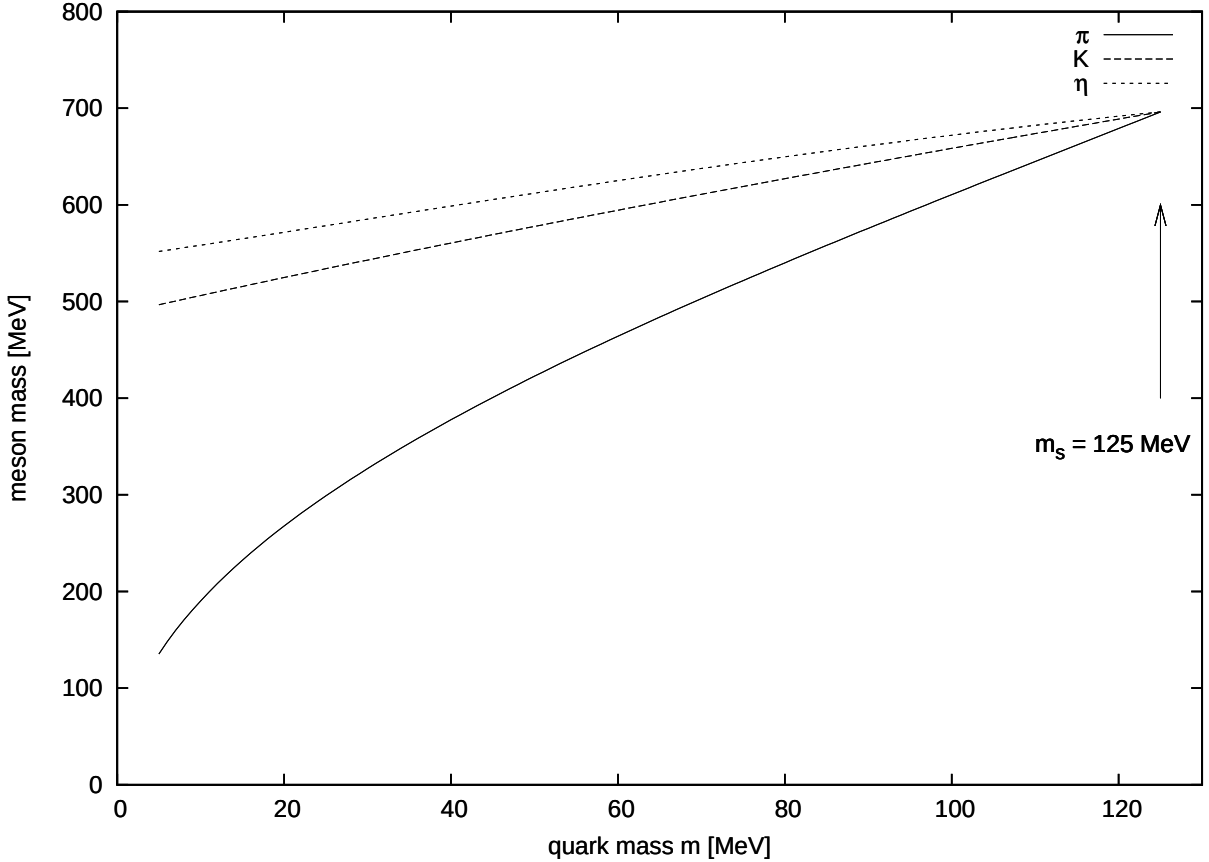
meson masses:

$$\begin{aligned} m_\pi^2 &= 2B_0 m, \\ m_K^2 &= B_0(m + m_s), \\ m_\eta^2 &= \frac{2}{3}B_0(m + 2m_s) \end{aligned}$$

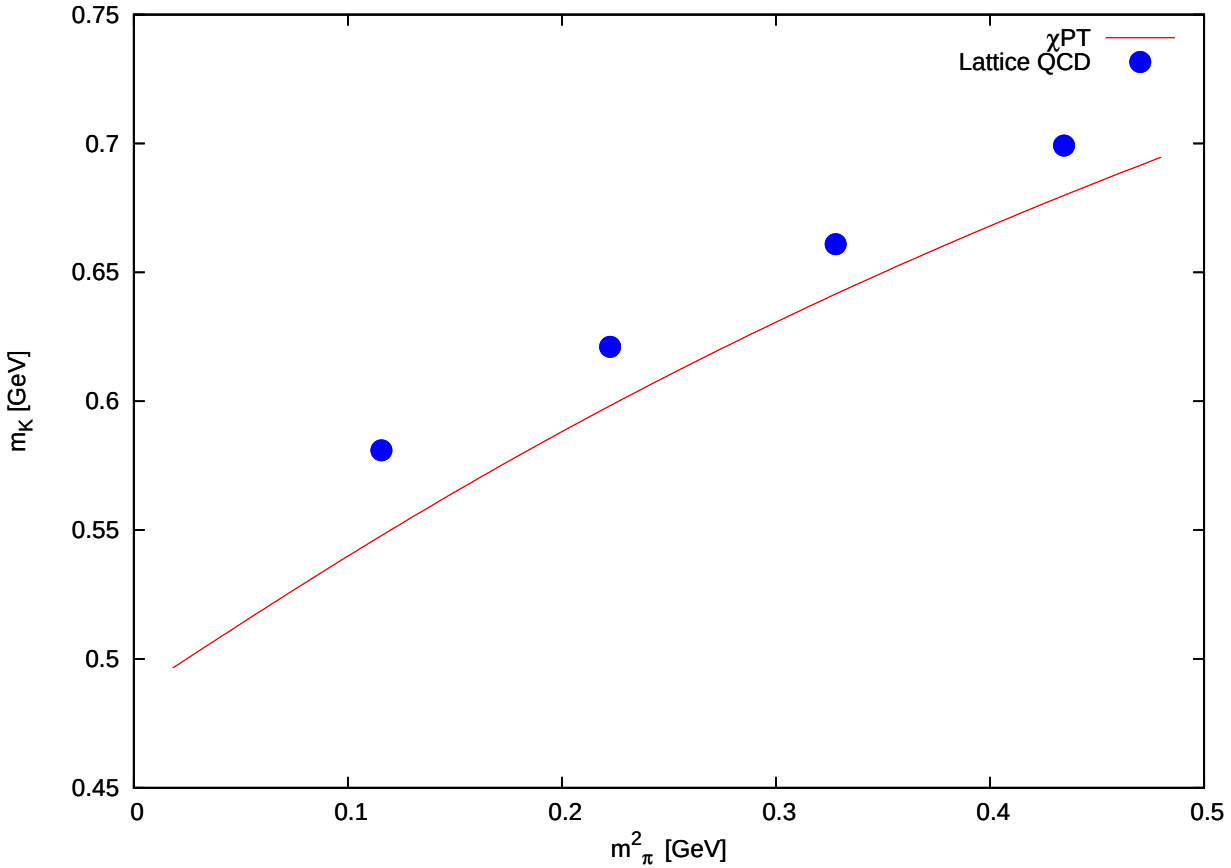
Gell-Mann-Okubo relation

$$3m_\eta^2 = 4m_K^2 - m_\pi^2$$

Quark mass dependence of the meson masses

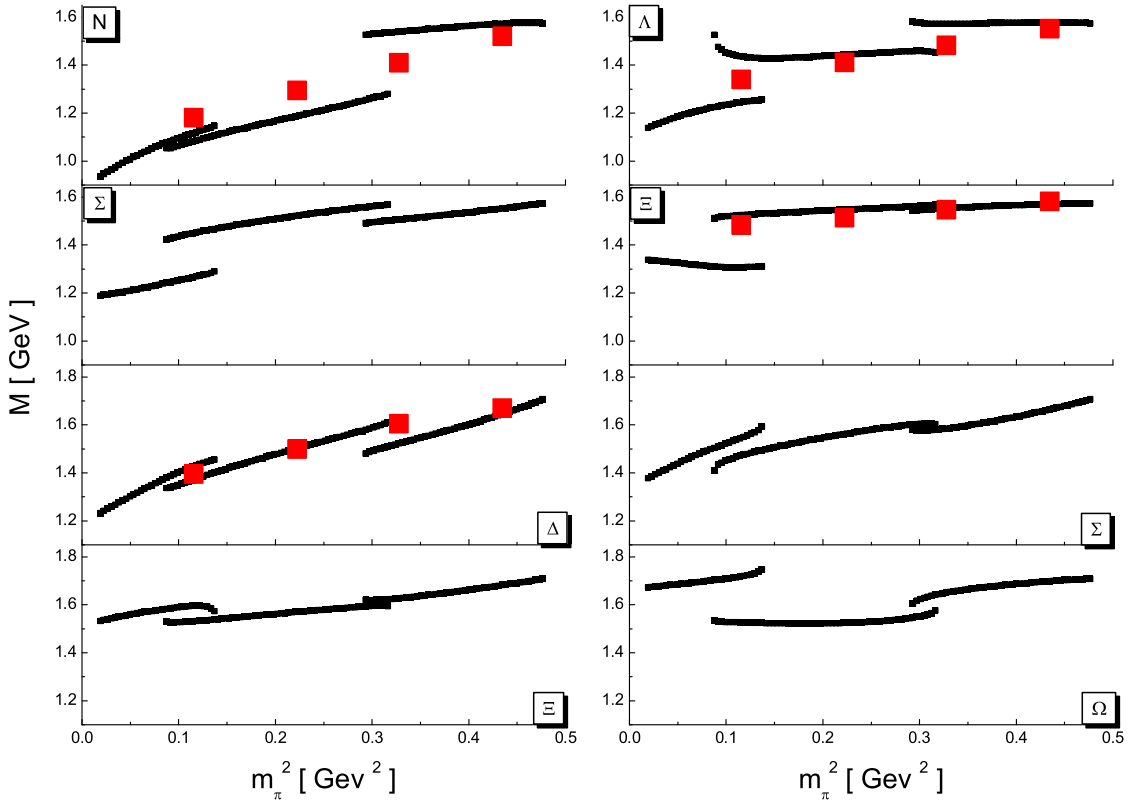


Lattice data vs. chiral extrapolation



Discontinous quark-mass dependence of baryon octet and decuplet masses

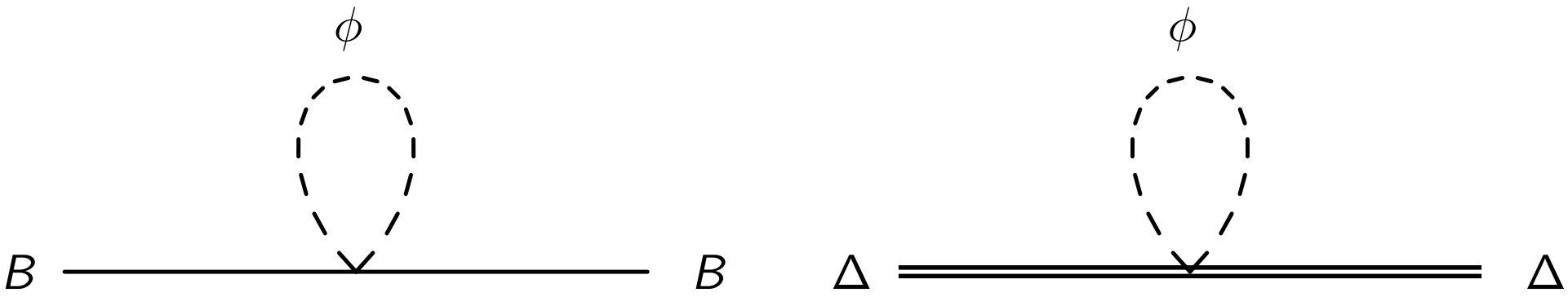
A.Semke and M.F.M. Lutz, nucl-th/0606027



Artifact or not ?

Tadpole integrals

$$\overbrace{B(x)B(y)B(z)}\,\overbrace{B(z)\phi(z)}\,\phi^{\dagger}(z)$$



$$\overbrace{\phi\ \phi^{\dagger}} = \int \frac{d^4k}{(2\pi)^4}\,\frac{i}{k^2-m^2+i\epsilon}$$

Self-Energies

$$\Sigma_{B\in[8]}^{tadpole}(p) = \frac{1}{(2f)^2}\sum_{Q\in[8]}\left[-m_Q^2 G_{BQ}^{(S)} - m_Q^2 \frac{\not{p}}{4} G_{BQ}^{(V)} + G_{BQ}^{(\chi,2)}\right]\frac{m_Q^2}{16\pi^2}\ln\left(\frac{m_Q^2}{\mu^2}\right)$$

$$\Sigma_{B\in[10]}^{tadpole}(p) = -\frac{1}{(2f)^2}\sum_{Q\in[8]}\left[m_Q^2 G_{BQ}^{(S)} + m_Q^2 \frac{\not{p}}{4} G_{BQ}^{(V)} + G_{BQ}^{(\chi,2)}\right]\frac{m_Q^2}{16\pi^2}\ln\left(\frac{m_Q^2}{\mu^2}\right)$$

B	Q	$G_{BQ}^{(S)}$
Δ	π	$\frac{1}{2}\left(12f_1^{(S)}+3f_2^{(S)}+4f_3^{(S)}+f_4^{(S)}+f_5^{(S)}+4f_6^{(S)}+f_7^{(S)}+f_8^{(S)}\right)$
	K	$\frac{1}{2}\left(16f_1^{(S)}+4f_2^{(S)}+4f_3^{(S)}+f_4^{(S)}+f_5^{(S)}+4f_6^{(S)}+f_7^{(S)}+f_8^{(S)}\right)$
	η	$\frac{1}{6}\left(12f_1^{(S)}+3f_2^{(S)}+8f_3^{(S)}+2f_4^{(S)}+2f_5^{(S)}\right)$
$\Sigma_{[10]}$	π	$\frac{1}{4}\left(24f_1^{(S)}+6f_2^{(S)}+12f_3^{(S)}+3f_4^{(S)}+3f_5^{(S)}+8f_6^{(S)}+2f_7^{(S)}+2f_8^{(S)}\right)$
	K	$\frac{1}{2}\left(16f_1^{(S)}+4f_2^{(S)}+4f_3^{(S)}+f_4^{(S)}+f_5^{(S)}+4f_6^{(S)}+f_7^{(S)}+f_8^{(S)}\right)$
	η	$\frac{1}{12}\left(24f_1^{(S)}+6f_2^{(S)}+4f_3^{(S)}+f_4^{(S)}+f_5^{(S)}\right)$
$\Xi_{[10]}$	π	$\frac{1}{12}\left(72f_1^{(S)}+18f_2^{(S)}+24f_3^{(S)}+6f_4^{(S)}+6f_5^{(S)}+20f_6^{(S)}+5f_7^{(S)}+5f_8^{(S)}\right)$
	K	$\frac{1}{3}\left(24f_1^{(S)}+6f_2^{(S)}+8f_3^{(S)}+2f_4^{(S)}+2f_5^{(S)}+4f_6^{(S)}+f_7^{(S)}+f_8^{(S)}\right)$
	η	$\frac{1}{12}\left(24f_1^{(S)}+6f_2^{(S)}+8f_3^{(S)}+2f_4^{(S)}+2f_5^{(S)}+12f_6^{(S)}+3f_7^{(S)}+3f_8^{(S)}\right)$
Ω	π	$\frac{1}{4}\left(24f_1^{(S)}+6f_2^{(S)}+4f_3^{(S)}+f_4^{(S)}+f_5^{(S)}+4f_6^{(S)}+f_7^{(S)}+f_8^{(S)}\right)$
	K	$\frac{1}{6}\left(48f_1^{(S)}+12f_2^{(S)}+20f_3^{(S)}+5f_4^{(S)}+5f_5^{(S)}+12f_6^{(S)}+3f_7^{(S)}+3f_8^{(S)}\right)$
	η	$\frac{1}{4}\left(8f_1^{(S)}+2f_2^{(S)}+4f_3^{(S)}+f_4^{(S)}+f_5^{(S)}+4f_6^{(S)}+f_7^{(S)}+f_8^{(S)}\right)$

Summary:

- We investigated the quark-mass dependence of mesons and baryons.
- We calculated contributions of tadpole diagrams to the self-energy.
- Problem of higher order corrections - a lot of new unknown couplings.
- How to fix these couplings constants? Use large N_c counting.