Stellar and extragalactic astronomy I - Problem Set 5
16.12.2010, Thursday, 10.45-12.15

1. **Dark matter in clusters of galaxies.** In most clusters of galaxies, the intra-cluster gas appears to be in approximate hydrostatic equilibrium.

a) Assuming spherical symmetry, show that anisotropy free Jeans equation can be written as follows

$$\frac{d\ln\rho_g}{d\ln r} + \frac{d\ln T}{d\ln r} = -\frac{GM(r)m}{kT}, \tag{1}$$

where ρ_g is the gas density, T is the gas temperature and m is the average mass of the gas particles (usually this mass is expressed in terms of the proton mass $m = \mu m_p$). We assume that the distribution of gas is isothermal thereafter.

b) Show that if dark matter distribution is isothermal and $\rho_g \ll \rho_{\rm DM}$, then

$$\frac{d\ln\rho_g}{d\ln r} = \beta\frac{d\ln\rho_{\rm DM}}{d\ln r} \quad \text{where} \quad \beta = \frac{\mu m_p\sigma^2}{kT}, \tag{2}$$

and σ is velocity dispersion of dark matter particles (do not confuse β with anisotropy parameter).

c) The intra-cluster gas is very hot and hence emits X-rays in the form of thermal bremsstrahlung. This radiation is observed by X-ray observatories as *Chandra* and *XMM-Newton*. In Fig. 1 relation between dark matter velocity dispersion and measured temperature of intra-cluster gas, for a sample of galaxy clusters and galaxy groups, was shown. Think through how these quantities were determined. Let us assume that gas is consisting mainly of ionized

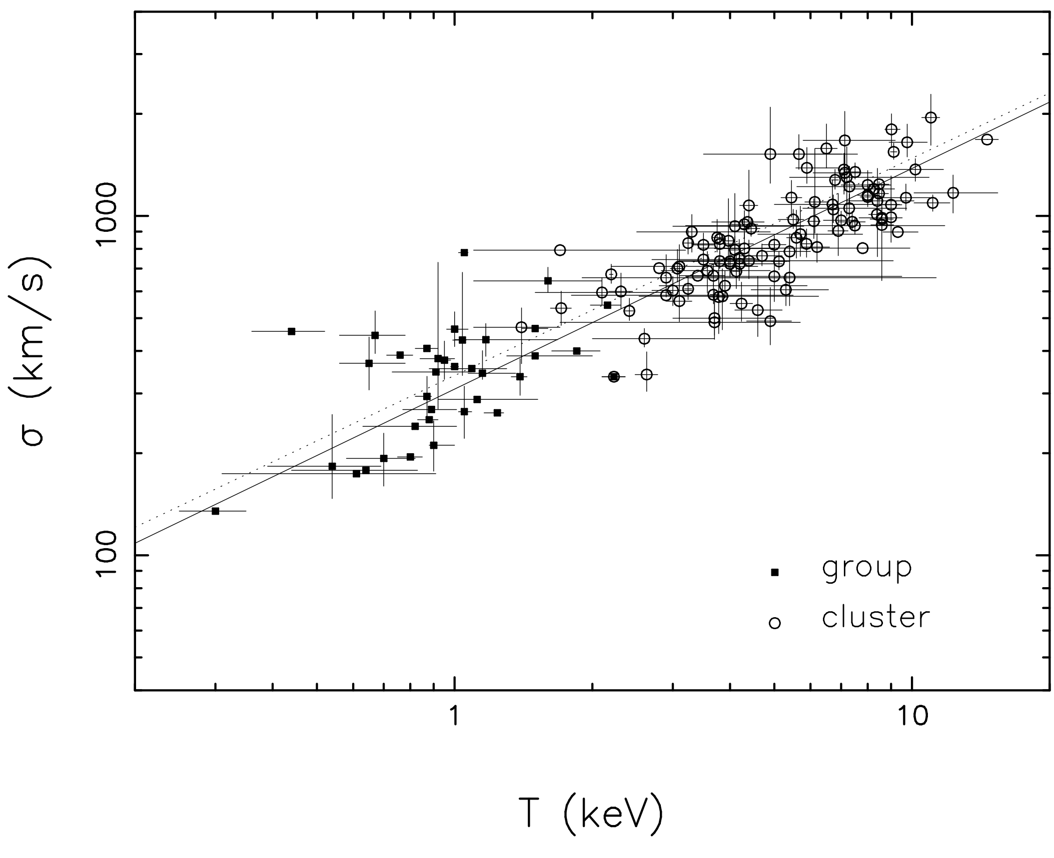

FIG. 1. Relation between velocity dispersion and temperature (Figure taken from Xue, Y., and X. Wu, 2000, Astroph. J. **538**, 65).

hydrogen atoms. Based on this estimate value of β parameter.

d) For small r, distribution of dark matter for isothermal sphere model is

$$\rho_{\rm DM}(r) \propto \frac{1}{(1+(r/r_c)^2)^{\frac{3}{2}}}, \tag{3}$$

where r_c is core radius (for large r we have $\rho(r) \propto r^{-2}$, as found for SIS solution). Show that corresponding gas density profile is the following

$$\rho_g(r) \propto \frac{1}{(1+(r/r_c)^2)^{\frac{3}{2}\beta}}. \tag{4}$$

The X-ray surface brightness distribution is proportional to the integral of $\rho_g^2(r)$ along the line of sight. Show that surface brightness for the gas distribution (4) has the functional form

$$S(R) = \frac{S(0)}{(1+(R/r_c)^2)^{3\beta-\frac{1}{2}}}, \tag{5}$$

where $S(0)$ is the central surface brightness and R is the projected radial distance from the cluster center on the sky plane. Assume the value of parameter β found previously. Based on X- ray surface brightness profile shown in Fig. 2, estimate value of the core radius r_c.

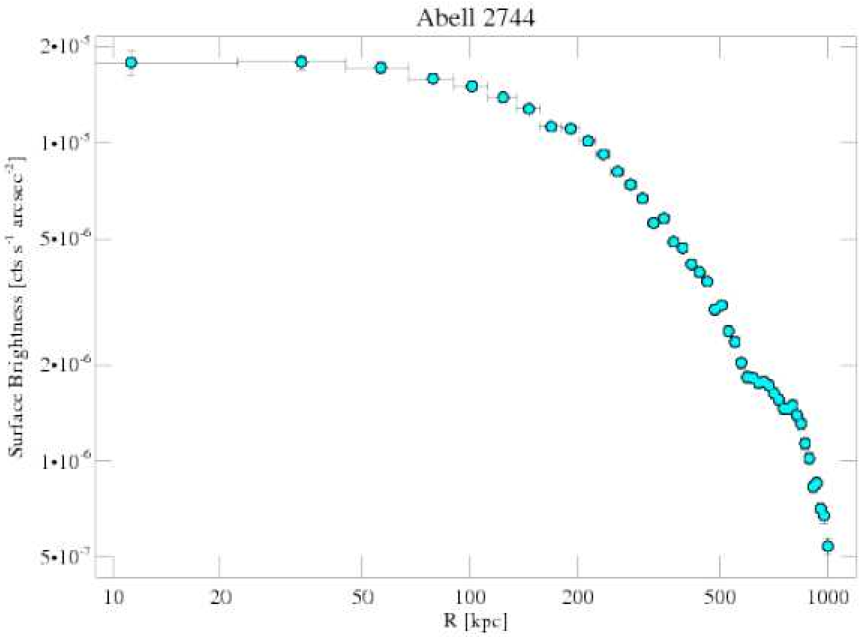

FIG. 2. X-ray surface brightness profile for Abell 2744 galaxy cluster (Figure taken from http://www.pa.msu.edu/astro/MC2/accept/).

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