Cosmological perturbations with holonomy corrections

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- J. M. , Thomas Cailleteau, Julien Grain and Aurelien Barrau, *Inflation in loop quantum cosmology: Dynamics and spectrum of gravitational waves*, Phys. Rev. D **81** (2010) 104049 [arXiv:1003.4660 [gr-qc]].
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Metric perturbations

The metric perturbations can be decomposed according to their spin as follows

$$g_{\mu\nu} = \underbrace{a^2 \begin{pmatrix} -1 & 0 \\ 0 & \delta_{ab} \end{pmatrix}}_{FRW\ \ k=0\ \ background} + \underbrace{a^2 \begin{pmatrix} -2\phi & \partial_a B \\ \partial_a B & -2\psi\delta_{ab} + \partial_a\partial_b E \end{pmatrix}}_{scalar\ \ (s=0)}$$

$$+ \underbrace{a^2 \begin{pmatrix} 0 & S_a \\ S_a & F_{a,b} + F_{b,a} \end{pmatrix}}_{vector\ \ (s=1)} + \underbrace{a^2 \begin{pmatrix} 0 & 0 \\ 0 & h_{ab} \end{pmatrix}}_{tensor\ \ (s=2)}.$$

Furthermore, the perturbation variables fulfill the following conditions:

$$\partial^a h_{ab} = 0 \ \ (transverse) \ \ \text{and} \ \ \delta^{ab} h_{ab} = 0 \ \ (traceless),$$

$$F^a{}_{,a} = 0 \ \ (divergence\ \ free) \ \ \text{and} \ \ S^a{}_{,a} = 0 \ \ (divergence\ \ free).$$

Preliminaries

The basic variable are decomposed for the background and perturbation parts:

$$K^i_a = \bar{k}\delta^i_a + \delta K^i_a, \quad E^a_i = \bar{p}\delta^a_i + \delta E^a_i, \text{(gravity)}$$
$$\varphi = \bar{\varphi} + \delta\varphi, \quad \pi = \bar{\pi} + \delta\pi, \text{(matter)}$$

such that $|\delta K^i_a/\bar{k}| \ll 1$, $|\delta E^a_i/\bar{p}| \ll 1$, $|\delta\varphi/\bar{\varphi}| \ll 1$ and $|\delta\pi/\bar{\pi}| \ll 1$.

Poisson bracket:

$$\{\cdot,\cdot\} = \{\cdot,\cdot\}_{\bar{k},\bar{p}} + \{\cdot,\cdot\}_{\delta K,\delta E} + \{\cdot,\cdot\}_{\bar{\varphi},\bar{\pi}} + \{\cdot,\cdot\}_{\delta\varphi,\delta\pi}.$$

The constraints $\mathcal{C}_I$ are subject of perturbative expansion:

$$\mathcal{C}_I = \mathcal{C}^{(0)}_I + \underbrace{\mathcal{C}^{(1)}_I}_{=0} + \mathcal{C}^{(2)}_I + \ldots$$

Holonomy corrections

are introduced by the replacement

$$\bar{k} \to \mathbb{K}[n] := \frac{\sin(n\bar{\mu}\gamma\bar{k})}{n\bar{\mu}\gamma},$$

in the classical constraints, where $n \in \mathbb{Z}$. In general $\bar{\mu} \propto \bar{p}^{\beta}$, where $-1/2 \leq \beta \leq 0$. For the so-called $\bar{\mu}-$scheme: $\bar{\mu} = \sqrt{\frac{\Delta}{\bar{p}}}$, where $\Delta = 2\sqrt{3}\pi\gamma l_{\text{Pl}}^2$ is the area gap derived from LQG.

Anomalies

Because the constraints are quantum-modified ($\mathcal{C}_I \to \mathcal{C}_I^Q$), there is a worry that the corresponding Poisson algebra will not be closed:

$$\{\mathcal{C}_I^Q, \mathcal{C}_J^Q\} = f^K{}_{IJ}(A_b^j, E_i^a)\mathcal{C}_K^Q + \mathcal{A}_{IJ}.$$

For consistency (closure of algebra), $\mathcal{A}_{IJ}$ is required to vanish.

Algebra of constraints

The Hamiltonian is a total constraint which is vanishing for all multiplier functions (N^i, N^a, N). Because $H[N^i, N^a, N] \approx 0$, then also the time derivative $\dot{H}[N^i, N^a, N] \approx 0$. Therefore, based on the Hamilton equation, one obtain

$$\left\{ H[N^i, N^a, N], H[M^i, M^a, M] \right\} \approx 0, \tag{1}$$

what explicitly writing is

$$\left\{ G[N^i] + D_{\mathrm{G}}[N^a] + H_{\mathrm{G}}[N], G[M^i] + D_{\mathrm{G}}[M^a] + H_{\mathrm{G}}[M] \right\} \approx 0.$$

Decomposing the Poisson bracket, one can directly find that condition (1) is fulfilled if the smeared constraints fulfill the first class algebra

$$\{\mathcal{C}_I, \mathcal{C}_J\} = f^K{}_{IJ}(A^j_b, E^a_i)\mathcal{C}_K.$$

Here, $f^K{}_{IJ}(A^j_b, E^a_i)$ are structure functions.

SCALAR PERTURBATIONS

The holonomy-modified Hamiltonian constraint can be written as:

$$H_G^Q[N] = \frac{1}{2\kappa}\int_\Sigma d^3x \left[\bar{N}(\mathcal{H}_G^{(0)} + \mathcal{H}_G^{(2)}) + \delta N \mathcal{H}_G^{(1)}\right], \text{ where}$$

$$\begin{aligned}
\mathcal{H}_G^{(0)} &= -6\sqrt{\bar{p}}(\mathbb{K}[1])^2, \\
\mathcal{H}_G^{(1)} &= -4\sqrt{\bar{p}}\left(\mathbb{K}[s_1] + \alpha_1\right)\delta_j^c \delta K_c^j - \frac{1}{\sqrt{\bar{p}}}\left(\mathbb{K}[1]^2 + \alpha_2\right)\delta_c^j \delta E_j^c \\
&+ \frac{2}{\sqrt{\bar{p}}}(1+\alpha_3)\partial_c \partial^j \delta E_j^c, \\
\mathcal{H}_G^{(2)} &= \sqrt{\bar{p}}(1+\alpha_4)\delta K_c^j \delta K_d^k \delta_k^c \delta_j^d - \sqrt{\bar{p}}(1+\alpha_5)(\delta K_c^j \delta_j^c)^2 \\
&- \frac{2}{\sqrt{\bar{p}}}\left(\mathbb{K}[s_2] + \alpha_6\right)\delta E_j^c \delta K_c^j - \frac{1}{2\bar{p}^{3/2}}\left(\mathbb{K}[1]^2 + \alpha_7\right)\delta E_j^c \delta E_k^d \delta_c^k \delta_d^j \\
&+ \frac{1}{4\bar{p}^{3/2}}\left(\mathbb{K}[1]^2 + \alpha_8\right)(\delta E_j^c \delta_c^j)^2 \\
&- \frac{1}{2\bar{p}^{3/2}}(1+\alpha_9)\delta^{jk}(\partial_c \delta E_j^c)(\partial_d \delta E_k^d).
\end{aligned}$$

Here, $\alpha_i(\bar{p}, \bar{k})$ are the counter-terms ($\alpha_i(\bar{p}, \bar{k}) \to 0$ for $\bar{\mu} \to 0$).

Diffeomorphism constraint takes the classical form

$$D^Q[N^a] = \frac{1}{\kappa}\int_\Sigma d^3x\delta N^c\left[\bar{p}\partial_c(\delta^d_k\delta K^k_d) - \bar{p}(\partial_k\delta K^k_c) - \bar{k}\delta^k_c(\partial_d\delta E^d_k)\right].$$

The scalar matter diffeomorphism constraint is

$$D_M[N^a] = \int_\Sigma \delta N^a\bar{\pi}(\partial_a\delta\varphi).$$

The scalar matter Hamiltonian can be expressed as follows

$$H^Q_M[N] = S_M[\bar{N}] + S_M[\delta N],$$

where

$$H_M[\bar{N}] = \int_\Sigma d^3x\bar{N}\left[\left(\mathcal{H}^{(0)}_\pi + \mathcal{H}^{(0)}_\varphi\right) + \left(\mathcal{H}^{(2)}_\pi + \mathcal{H}^{(2)}_\nabla + \mathcal{H}^{(2)}_\varphi\right)\right],$$

$$H_M[\delta N] = \int_\Sigma d^3\delta N\left[\mathcal{H}^{(1)}_\pi + \mathcal{H}^{(1)}_\varphi\right].$$

$$\mathcal{H}^{(0)}_{\pi} = \frac{\bar{\pi}^2}{2\bar{p}^{3/2}} \qquad \mathcal{H}^{(1)}_{\pi} = \frac{\bar{\pi}\delta\pi}{\bar{p}^{3/2}} - \frac{\bar{\pi}^2}{2\bar{p}^{3/2}}\frac{\delta^j_c \delta E^c_j}{2\bar{p}}$$

$$\mathcal{H}^{(0)}_{\varphi} = \bar{p}^{3/2} V(\bar{\varphi}) \qquad \mathcal{H}^{(1)}_{\varphi} = \bar{p}^{3/2}\left[V_{,\varphi}(\bar{\varphi})\delta\varphi + V(\bar{\varphi})\frac{\delta^j_c \delta E^c_j}{2\bar{p}}\right]$$

$$\mathcal{H}^{(2)}_{\pi} = \frac{1}{2}\frac{\delta\pi^2}{\bar{p}^{3/2}} - \frac{\bar{\pi}\delta\pi}{\bar{p}^{3/2}}\frac{\delta^j_c \delta E^c_j}{2\bar{p}} + \frac{1}{2}\frac{\bar{\pi}^2}{\bar{p}^{3/2}}\left[\frac{(\delta^j_c \delta E^c_j)^2}{8\bar{p}^2} + \frac{\delta^k_c \delta^j_d \delta E^c_j \delta E^d_k}{4\bar{p}^2}\right],$$

$$\mathcal{H}^{(2)}_{\nabla} = \frac{1}{2}\sqrt{\bar{p}}(1+\alpha_{10})\delta^{ab}\partial_a\delta\varphi\partial_b\delta\varphi,$$

$$\mathcal{H}^{(2)}_{\varphi} = \frac{1}{2}\bar{p}^{3/2}V_{,\varphi\varphi}(\bar{\varphi})\delta\varphi^2 + \bar{p}^{3/2}V_{,\varphi}(\bar{\varphi})\delta\varphi\frac{\delta^j_c \delta E^c_j}{2\bar{p}}$$
$$+ \bar{p}^{3/2}V(\bar{\varphi})\left[\frac{(\delta^j_c \delta E^c_j)^2}{8\bar{p}^2} - \frac{\delta^k_c \delta^j_d \delta E^c_j \delta E^d_k}{4\bar{p}^2}\right].$$

Brackets

The total Hamiltonian and diffeomorphism constraints are:

$$
\begin{aligned}
H_{tot}[N] &= H_G^Q[N] + H_M^Q[N], \\
D_{tot}[N^a] &= D_G[N^a] + D_M[N^a].
\end{aligned}
$$

The Poisson bracket between two total diffeomorphism constraints is vanishing:

$$\{D_{tot}[N_1^a], D_{tot}[N_2^a]\} = 0.$$

The bracket between the total Hamiltonian and diffeomorphism constraints can be decomposed as follows:

$$
\begin{aligned}
\{H_{tot}[N], D_{tot}[N^a]\} &= \left\{H_M^Q[N], D_{tot}[N^a]\right\} + \left\{H_G^Q[N], D_G[N^a]\right\} \\
&+ \left\{H_G^Q[N], D_M[N^a]\right\}.
\end{aligned}
$$

$$\left\{H_M^Q[N], D_{tot}[N^a]\right\} = -H_M^Q[\delta N^a \partial_a \delta N].$$

$$\left\{H_G^Q[N], D_G[N^a]\right\} = -H_G^Q\left[\delta N^a \partial_a \delta N\right] + \mathcal{B}\ D_G[N^a] + \frac{\sqrt{\bar{p}}}{\kappa}\int_\Sigma d^3x \delta N^a (\partial_a \delta N)\mathcal{A}_1 + \frac{\bar{N}\sqrt{\bar{p}}\bar{k}}{\kappa}\int_\Sigma d^3x \delta N^a (\partial_i \delta K_a^i)\mathcal{A}_2 + \frac{\bar{N}}{\kappa\sqrt{\bar{p}}}\int_\Sigma d^3x \delta N^i (\partial_a \delta E_i^a)\mathcal{A}_3 + \frac{\bar{N}}{2\kappa\sqrt{\bar{p}}}\int_\Sigma d^3x (\partial_a \delta N^a)(\delta E_i^b \delta_b^i)\mathcal{A}_4.$$

The functions $\mathcal{A}_1, \ldots, \mathcal{A}_4$ are the first anomalies coming from the effective nature of the Hamiltonian constraint.

$$\left\{H_G^Q[N], D_M[N^a]\right\} = 0.$$

The Poisson bracket between the two total Hamiltonian constraints can be decomposed in the following way:

$$
\begin{aligned}
\{H_{tot}[N_1], H_{tot}[N_2]\} &= \left\{H^Q_G[N_1], H^Q_G[N_2]\right\} + \{H_M[N_1], H_M[N_2]\} \\
&+ \left[\left\{H^Q_G[N_1], H_M[N_2]\right\} - (N_1 \leftrightarrow N_2)\right].
\end{aligned}
$$

Here, the Poisson bracket between two matter Hamiltonians is

$$
\left\{H^Q_M[N_1], H^Q_M[N_2]\right\} = (1+\alpha_{10}) D_M \left[\frac{\bar{N}}{\bar{p}} \partial^a (\delta N_2 - \delta N_1)\right].
$$

The appearance of the front-factor $(1+\alpha_{10})$ will allow us to close the algebra of total constraints.

$$\left\{H_G^Q[N_1], H_G^Q[N_2]\right\} = (1+\alpha_3)(1+\alpha_5)D_G\left[\frac{\bar{N}}{\bar{p}}\partial^a(\delta N_2 - \delta N_1)\right]$$
$$+\frac{\bar{N}}{\kappa}\int_\Sigma d^3x\partial^a(\delta N_2 - \delta N_1)(\partial_i\delta K_a^i)(1+\alpha_3)\mathcal{A}_5$$
$$+\frac{\bar{N}}{\kappa\bar{p}}\int_\Sigma d^3x(\delta N_2 - \delta N_1)(\partial^i\partial_a\delta E_i^a)\mathcal{A}_6$$
$$+\frac{\bar{N}}{\kappa}\int_\Sigma d^3x(\delta N_2 - \delta N_1)(\delta_i^a\delta K_a^i)\mathcal{A}_7$$
$$+\frac{\bar{N}}{\kappa\bar{p}}\int_\Sigma d^3x(\delta N_2 - \delta N_1)(\delta_a^i\delta E_i^a)\mathcal{A}_8$$

The $\mathcal{A}_5, \ldots, \mathcal{A}_8$ are the next four anomalies. Moreover, the diffeomorphism constraint is multiplied by the factor $(1+\alpha_3)(1+\alpha_5)$.

$$\left\{H_G^Q[N_1], H_M[N_2]\right\} - (N_1 \leftrightarrow N_2) =$$

$$= \frac{1}{2}\int_\Sigma d^3x \bar{N}(\delta N_2 - \delta N_1)\left(\frac{\bar{\pi}^2}{2\bar{p}^3} - V(\bar{\varphi})\right)(\partial_c\partial^j\delta E^c_j)\mathcal{A}_9$$
$$+3\int_\Sigma d^3x \bar{N}(\delta N_2 - \delta N_1)\left(\frac{\bar{\pi}\delta\pi}{\bar{p}^2} - \bar{p}V_\varphi(\bar{\varphi})\delta\varphi\right)\mathcal{A}_{10}$$
$$+\int_\Sigma d^3x \bar{N}(\delta N_2 - \delta N_1)(\delta^c_j\delta K^c_j)\left(\frac{\bar{\pi}^2}{2\bar{p}^3} - V(\bar{\varphi})\right)\bar{p}\mathcal{A}_{11}$$
$$+\frac{1}{2}\int_\Sigma d^3x \bar{N}(\delta N_2 - \delta N_1)(\delta^j_c\delta E^c_j)\left(\frac{\bar{\pi}^2}{2\bar{p}^3}\right)\mathcal{A}_{12}$$
$$+\frac{1}{2}\int_\Sigma d^3x \bar{N}(\delta N_2 - \delta N_1)(\delta^j_c\delta E^c_j)V(\bar{\varphi})\mathcal{A}_{13}$$

The functions $\mathcal{A}_9, \ldots, \mathcal{A}_{13}$ are the last five anomalies.

Anomaly freedom

The requirement of anomaly freedom is equivalent to the conditions $\mathcal{A}_i = 0$ for $i = 1, \ldots, 13$. Furthermore $(1+\alpha_3)(1+\alpha_5) = (1+\alpha_{10})$. These conditions uniquely determine form of the counter terms α_i for $i = 1, \ldots, 10$.

Moreover, we have

$$\begin{aligned} \mathcal{A}_7 &= 2(1+2\beta)(\Omega\mathbb{K}[1]^2 - \mathbb{K}[2]^2), \\ \mathcal{A}_8 &= \bar{k}(1+2\beta)(\mathbb{K}[2]^2 - \Omega\mathbb{K}[1]^2). \end{aligned}$$

The anomaly freedom conditions for those terms, $\mathcal{A}_7 = 0$ and $\mathcal{A}_8 = 0$, are fulfilled if and only if $\beta = -1/2$. The choice $\beta = -1/2$ is called the $\bar{\mu}-$scheme ('new quantization scheme').

Our result show that the $\bar{\mu}-$scheme is embedded in the structure of the theory and this gives a new motivation for this particular choice of quantization scheme.

The counter-terms allowing the algebra to be anomaly-free are uniquely determined, and are given by:

$$
\begin{aligned}
\alpha_1 &= \mathbb{K}[2] - \mathbb{K}[s_1], \\
\alpha_2 &= 2\mathbb{K}[1]^2 - 2\bar{k}\mathbb{K}[2], \\
\alpha_3 &= 0, \\
\alpha_4 &= \Omega - 1, \\
\alpha_5 &= \Omega - 1, \\
\alpha_6 &= 2\mathbb{K}[2] - \mathbb{K}[s_2] - \bar{k}\Omega, \\
\alpha_7 &= -4\mathbb{K}[1]^2 + 6\bar{k}\mathbb{K}[2] - 2\bar{k}^2\Omega, \\
\alpha_8 &= -4\mathbb{K}[1]^2 + 6\bar{k}\mathbb{K}[2] - 2\bar{k}^2\Omega, \\
\alpha_9 &= 0, \\
\alpha_{10} &= \Omega - 1,
\end{aligned}
$$

where

$$\Omega := \cos(2\bar{\mu}\gamma\bar{k}) = 1 - 2\frac{\rho}{\rho_c}.$$

Algebra of constraints:

$$\begin{aligned}
\{D_{tot}[N_1^a], D_{tot}[N_2^a]\} &= 0, \\
\{H_{tot}[N], D_{tot}[N^a]\} &= -H_{tot}[\delta N^a \partial_a \delta N], \\
\{H_{tot}[N_1], H_{tot}[N_2]\} &= \Omega D_{tot}\left[\frac{\bar{N}}{\bar{p}} \partial^a (\delta N_2 - \delta N_1)\right].
\end{aligned}$$

Although the algebra is closed, there is deformation with respect to the classical case, due to presence of the factor

$$\Omega = \cos(2\bar{\mu}\gamma\bar{k}) = 1 - 2\frac{\rho}{\rho_c} \in [-1, 1].$$

What is the interpretation? Classically, we have

$$\{H_{tot}[N_1], H_{tot}[N_2]\} = sD\left[\frac{\bar{N}}{\bar{p}} \partial^a (\delta N_2 - \delta N_1)\right],$$

where $s = 1$ corresponds to the Lorentzian signature and $s = -1$ to the Euclidean one.

- The effective algebra of constraints shows that the space is Euclidian for $\rho > \rho_c/2$. At the particular value when $\rho = \frac{\rho_c}{2}$, the geometry switches to the Minkowski one.
- It is interesting to notice that this model naturally have properties of the Hartle-Hawking no-boundary proposal.
- The similar effect observed also for the spherically symmetric models (see [1] for the recent discussion).

Lots of questions arise, e.g. :

- Is the sigh change only a perturbative effect?
- Is the background dynamics also modified?
- What with the standard picture of the bouncing cosmology?
- Is it a hint that the quantum algebra in LQG is also modified? ($[\hat{H}, \hat{H}] = \Omega \hat{D}$?)
- Is there relation to non-commutative geometry or DSR ?

[1] M. Bojowald and G. M. Paily, "Deformed General Relativity and Effective Actions from Loop Quantum Gravity," arXiv:1112.1899 [gr-qc].

Equations of motion - Longitudinal gauge ($E = 0 = B$)

We find

$$\ddot{\phi} + 2\left[\mathcal{H} - \left(\frac{\ddot{\bar{\varphi}}}{\dot{\bar{\varphi}}} + \epsilon\right)\right]\dot{\phi} + 2\left[\dot{\mathcal{H}} - \mathcal{H}\left(\frac{\ddot{\bar{\varphi}}}{\dot{\bar{\varphi}}} + \epsilon\right)\right]\phi - c_s^2\nabla^2\phi = 0,$$

with the quantum correction

$$\epsilon = \frac{1}{2}\frac{\dot{\Omega}}{\Omega} = 3\mathbb{K}[2]\left(\frac{\rho + P}{\rho_c - 2\rho}\right),$$

and the squared velocity $c_s^2 = \Omega$. The derived equation is the same as this found by E. Wilson-Ewing[2] in his approach. This non-trivial equivalence of both approaches may suggests uniqueness in defining theory of scalar perturbations with holonomy corrections in anomaly-free manner.

[2] E. Wilson-Ewing, "Holonomy Corrections in the Effective Equations for Scalar Mode Perturbations in Loop Quantum Cosmology," arXiv:1108.6265.

Equations of motion - Gauge-invariant variables

Gauge-invariant variables (modified Bardeen's potentials):

$$\begin{aligned}
\Phi &= \phi + \frac{1}{\Omega}(\dot{B} - \ddot{E}) + \left(\frac{\mathbb{K}[2]}{\Omega} - \frac{\dot{\Omega}}{\Omega}\right)(B - \dot{E}), \\
\Psi &= \psi - \frac{\mathbb{K}[2]}{\Omega}(B - \dot{E}), \\
\delta\varphi^{GI} &= \delta\varphi + \frac{\dot{\bar{\varphi}}}{\Omega}(B - \dot{E}).
\end{aligned}$$

The gauge invariant variables are modified since the very structure of spacetime is deformed.

The equations of motion for Φ and Ψ are the same as this found for the longitudinal gauge. Moreover

$$\delta\ddot{\varphi}^{GI} + 2\mathbb{K}[2]\delta\dot{\varphi}^{GI} - \Omega\nabla^2\delta\varphi^{GI} + \bar{p}V_{,\varphi\varphi}(\bar{\varphi})\delta\varphi^{GI} + 2\bar{p}V_{,\varphi}(\bar{\varphi})\Psi - 4\dot{\bar{\varphi}}^{GI}\dot{\Psi} = 0.$$

Considering the scalar perturbations, there is only one physical degree of freedom. This, so-called Mukhanov variable, combines both the perturbation of the metric and the perturbation of matter.

We define the analogous of the Mukhanov variable

$$v := \sqrt{\bar{p}} \left(\delta\varphi^{GI} + \frac{\dot{\bar{\varphi}}}{\mathbb{K}[2]} \Psi \right).$$

Modified Mukhanov equation:

$$\ddot{v} - \Omega \nabla^2 v - \frac{\ddot{z}}{z} v = 0,$$

$$z = \sqrt{\bar{p}} \frac{\dot{\bar{\varphi}}}{\mathbb{K}[2]}.$$

One can no define the perturbation of curvature $\mathcal{R}$ such that $\mathcal{R} = \frac{v}{z}$. Based on this, the power spectrum of scalar perturbations can be computed.

TENSOR PERTURBATIONS

The holonomy-corrected Hamiltonian constraint (old version)[3]:

$$
\begin{aligned}
H^Q_G[N] &= \frac{1}{2\kappa}\int_\Sigma d^3x \bar{N}\left[-6\sqrt{\bar{p}}\,(\mathbb{K}[1])^2 - \frac{1}{2\bar{p}^{3/2}}\,(\mathbb{K}[1])^2\,(\delta E^c_j \delta E^d_k \delta^k_c \delta^j_d)\right.\\
&+ \sqrt{\bar{p}}(\delta K^j_c \delta K^k_d \delta^c_k \delta^d_j) - \frac{2}{\sqrt{\bar{p}}}\,(\mathbb{K}[2])\,(\delta E^c_j \delta K^j_c)\\
&+ \left.\frac{1}{\bar{p}^{3/2}}(\delta_{cd}\delta^{jk}\delta^{ef}\partial_e E^c_j \partial_f E^d_k)\right].
\end{aligned}
$$

The Poisson bracket

$$\left\{H^Q_G[N_1], H^Q_G[N_2]\right\} = 0$$

is anomaly-free. Moreover, the diffeomorphism constraint is equal zero, because for the tensor modes $N^a = 0$. Therefore, other possible Poisson brackets are vanishing.

[3] M. Bojowald and G. M. Hossain, "Loop quantum gravity corrections to gravitational wave dispersion," Phys. Rev. D **77** (2008) 023508.

New Hamiltonian constraint for the tensor modes

By using the holonomy corrections found while considering the scalar perturbations we can redefine the Hamiltonian constraint for the tensor modes:

$$
\begin{aligned}
H_G^Q[N] &= \frac{1}{2\kappa}\int_\Sigma d^3x\bar{N}\Big[-6\sqrt{\bar{p}}\,(\mathbb{K}[1])^2 \\
&- \frac{1}{2\bar{p}^{3/2}}\left(6\bar{k}\mathbb{K}[2]-3\mathbb{K}[1]^2-2\bar{k}^2\Omega\right)\left(\delta E^c_j\delta E^d_k\delta^k_c\delta^j_d\right) \\
&+ \sqrt{\bar{p}}\Omega(\delta K^j_c\delta K^k_d\delta^c_k\delta^d_j)-\frac{2}{\sqrt{\bar{p}}}\left(2\mathbb{K}[2]-\bar{k}\Omega\right)\left(\delta E^c_j\delta K^j_c\right) \\
&+ \frac{1}{\bar{p}^{3/2}}(\delta_{cd}\delta^{jk}\delta^{ef}\partial_e E^c_j\partial_f E^d_k)\Big].
\end{aligned}
$$

Based on this we obtain the following equation of motion:

$$
\frac{d^2}{d\eta^2}h^i_a+2\left(\mathbb{K}[2]-\frac{1}{2\Omega}\frac{d\Omega}{d\eta}\right)\frac{d}{d\eta}h^i_a-\Omega\nabla^2h^i_a=0.
$$

Equation of motion and the power spectrum

With use of the *old* Hamiltonian one can derive

$$\frac{d^2}{d\eta^2}h^i_a + 2aH\frac{d}{d\eta}h^i_a - \nabla^2 h^i_a + m_Q^2 h^i_a = 0,$$

where $h^i_a = h_\oplus (e^\oplus)^i_a + h_\otimes (e^\otimes)^i_a$. The quantum gravitationally induced *effective mass* is given by

$$m_Q^2 := 16\pi G a^2 \frac{\rho}{\rho_c}\left(\frac{2}{3}\rho - V\right).$$

For convenience we introduce the variable u and perform its Fourier decomposition as follows:

$$\frac{ah_\oplus}{\sqrt{16\pi G}} = \frac{ah_\otimes}{\sqrt{16\pi G}} = u(\mathbf{x},\eta) = \int \frac{d^3k}{(2\pi)^3} u_{\mathbf{k}}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}},$$

such that

$$\frac{d^2}{d\eta^2}u_{\mathbf{k}}(\eta) + \left[k^2 + m^2_{\text{eff}}\right] u_{\mathbf{k}}(\eta) = 0, \quad \text{where} \quad m^2_{\text{eff}} = m_Q^2 - \frac{a''}{a}.$$

Quantization

The quantization of the field u follows the canonical procedure. The Fourier components $\hat{u}_{\mathbf{k}}$ are promoted to the quantum operators:

$$\hat{u}_{\mathbf{k}}(\eta) = f_k(\eta)\hat{b}_{\mathbf{k}} + \overline{f_k}(\eta)\hat{b}^{\dagger}_{-\mathbf{k}},$$

where $f_k(\eta)$ is the so-called mode function which satisfies the same equation as $u_{\mathbf{k}}(\eta)$ and the Wronskian condition $\overline{f_k}f_k' - f_k\overline{f_k'} = -i$.

The correlation function of the $\hat{h}^i_a$ field is defined as follows

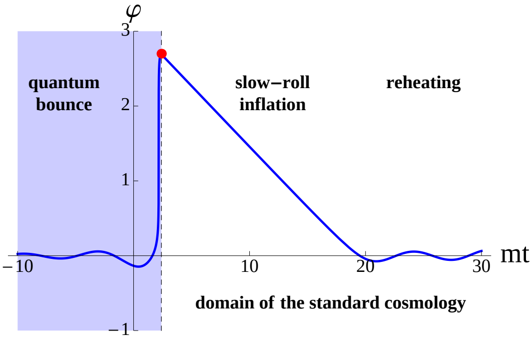
$$\langle 0|\hat{h}^a_b(\mathbf{x},\eta)\hat{h}^b_a(\mathbf{y},\eta)|0\rangle = \int_0^{\infty}\frac{dk}{k}\mathcal{P}_{\mathrm{T}}(k,\eta)\frac{\sin kr}{kr},$$

where $r = |\mathbf{x} - \mathbf{y}|$ and the tensor power spectrum

$$\mathcal{P}_{\mathrm{T}}(k,\eta) = \frac{64\pi G}{a^2(\eta)}\frac{k^3}{2\pi^2}|f_k(\eta)|^2.$$

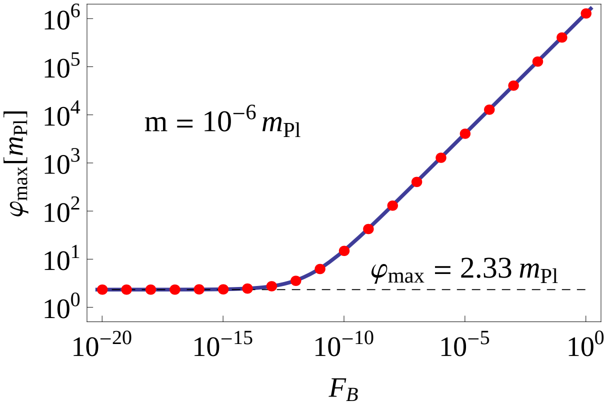
Background evolution: Inflation in LQC

We consider a model with the massive scalar field, $V(\varphi) = \frac{1}{2}m^2\varphi^2$.



Equations of motion for the background are:

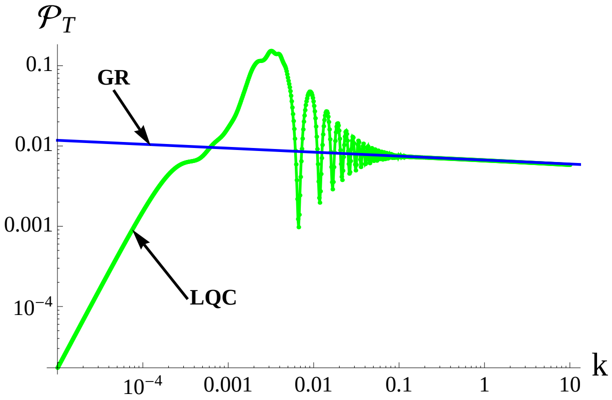
$$H^2 = \frac{\kappa}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right) \quad \text{and} \quad \ddot{\varphi} + 3H\dot{\varphi} + m^2\varphi = 0.$$



$$F_B := \frac{V(\varphi_B)}{\rho_c} \in [0, 1]$$

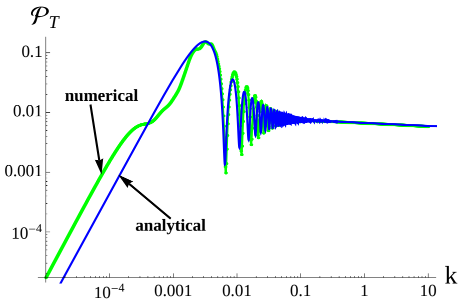
The corresponding minimal number of e-folds $N_{\rm min} \approx 34$, assuming that $\rho_c = \frac{\sqrt{3}}{16\pi^2\gamma^3} m_{\rm Pl}^4 \simeq 0.82 m_{\rm Pl}^4$.

Tensor power spectrum



In the IR region the LQC-spectrum behaves as $\mathcal{P}_\mathrm{T} \propto k^2$ while in the UV region $\mathcal{P}_\mathrm{T} \propto k^{-2\epsilon}$, where $\epsilon \ll 1$ is the slow-roll parameter. Here $m = 10^{-2} m_\mathrm{Pl}$. As the initial condition we used Minkowski vacuum ($f_k = e^{-ik\eta}/\sqrt{2k}$), at the scales much shorter that the Hubble radius.

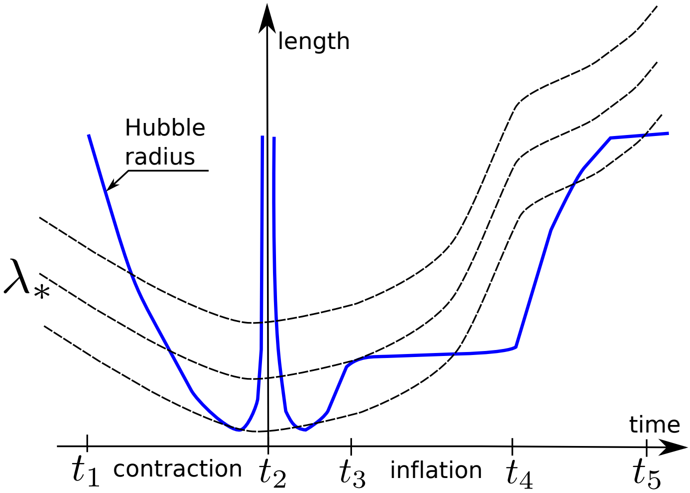
Analytical approximation of the numerical results



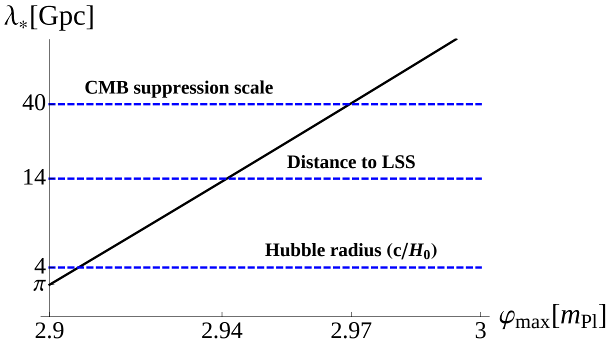
We used matching conditions for the mode functions, between contraction, bounce and slow-roll inflation, obtaining

$$\mathcal{P}_{\mathrm{T}}(k) = \frac{16}{\pi}\left(\frac{H}{m_{\mathrm{Pl}}}\right)^2\left(\frac{k}{aH}\right)^{-2\epsilon}|\alpha_k - \beta_k|^2,$$

where α_k and β_k are Bogoliubov coefficients.



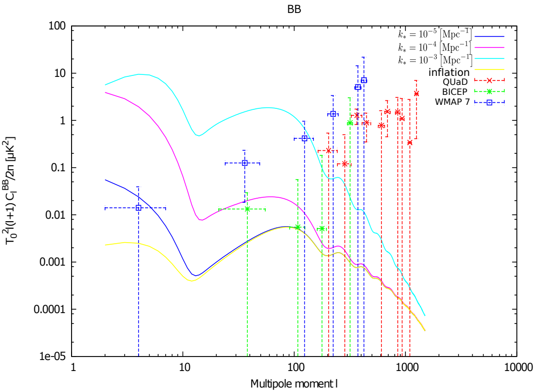
Scale of the *bump* in the power spectrum $k_* \sim \frac{1}{\lambda_*}$.



The present value of λ_* is very sensitive on duration of inflationary phase[4]. It makes possibility of observing the suppression due to the bounce almost improbable. There is only very narrow observational window for kinetic bounces with $F_B \sim 10^{-13}$.

[4] J. M. , M. Kamionka, A. Kurek and M. Szydlowski, "Observational hints on the Big Bounce," JCAP **1007** (2010) 004 [arXiv:1005.0814 [gr-qc]].

B-type polarization spectrum of the CMB



It was assumed that inflation mass $m = 10^{-6} m_{\mathrm{Pl}}$. Based on the present observational bounds on the BB spectrum, we find the preliminary constraint

$$F_B > 2.3 \cdot 10^{-13} \quad (\textit{Big Bounce is not symmetric}).$$

Observational perspectives

- Quantum corrections during inflation ($\mathcal{O}(V/\rho_c) \sim 10^{-12}$) are undetectable.
- The possibility of observing suppression and bump in the power spectrum is very sensitive on the initial conditions. Only tiny observational window in the parameter space.
- The oscillations in the power spectrum give a chance of observing some footprints of the Planck epoch. However, this effect is weak and may be undetectable due to the cosmic variance.
- It is possible that the anomalous behavior of the TT spectrum of the CMB at $l \approx 20$ and $l \approx 40$ can be related with oscillations in the power spectrum.
- A glimpse of hope - some new type non-Gaussian effects.
- No-go for testing Big Bounce? All relevant information above horizon, therefore inaccessible.

Outlook

- Understanding of the transition from Lorentzian to Euclidian domain.
- The modified Mukhanov equation can be directly applied to compute power spectrum of the scalar perturbations with the holonomy corrections.
- The issue of initial conditions (matching conditions) for the perturbations at $\rho = \rho_c/2$ ($\Omega = 0$). Maybe scale-invariant spectrum without inflation?
- Holonomy corrections to the inflationary spectrum during inflation (A_S, A_T, n_S, n_T, r). Unfortunately very tiny: $\mathcal{O}(V/\rho_c) \sim 10^{-12}$.
- Comparison with the CMB data (TT, TE, EE and BB spectra). However, no breakthrough is expected. But, only some weak constraints as e.g. $\gamma < 1100$ found for the Barbero-Immirzi parameter.