Tensor power spectrum with holonomy corrections in loop quantum cosmology

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In this paper we consider tensor perturbations produced at a bounce phase in the presence of holonomy corrections. Here, the bounce phase and holonomy corrections originate from loop quantum cosmology. We rederive formulas for the corrections of a model with scalar field content. Background dynamics with a free scalar field and multifluid potential are considered. Because the considerations are semiclassical, effects of quantum fluctuations of the background dynamics are not taken into account. Quantum and classical backreaction effects are also neglected. To find the spectrum of the gravitational waves, both analytical approximations and numerical investigations are performed. We have found analytical solutions on superhorizontal and subhorizontal regimes and derived the corresponding tensor power spectra. Also, the occupation number $n_{\mathbf{k}}$ and the parameter $\Omega_{\rm gw}$ were derived in the subhorizontal limit, leading to the extremely low present value of $\Omega_{\rm gw}$. The final results are the numerical power spectra of the gravitational waves produced in the presence of quantum holonomy corrections. The spectrum obtained has two UV and IR branches where $\mathcal{P}_T \propto k^2$; however, they have different prefactors. The spectrum connecting these regions is in the form of oscillations. We have found good agreement between the numerical spectrum and the spectrum obtained from the analytical model. The obtained spectrum can be directly applied as an initial condition for the inflationary modes. Based on our results, we discuss implications on the CMB radiation, in particular, on B-type polarization.

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I. INTRODUCTION

In the Minkowski background free gravitational waves fulfill the wave equation $(\partial_t^2 - \nabla^2)h_{\mu\nu} = 0$. Solutions of this equation are plane waves, $h_{\mu\nu} = P_{\mu\nu}e^{i(\mathbf{k}\cdot\mathbf{x}-|\mathbf{k}|t)}$; here $P_{\mu\nu}$ is the polarization tensor. However, when cosmological expansion is turned on [we assume a flat Friedmann-Robertson-Walker (FRW) background here] additional terms appear and the equation of motion is modified to $(\partial_t^2 + 3H\partial_t - \nabla^2)h_{\mu\nu} = 0$, where H is the Hubble parameter. We see that the cosmological term acts as effective friction. When the Universe undergoes expansion, $H > 0$ and gravitational waves are damped. This situation corresponds to the present stage of evolution. However, when the Universe is in the contracting phase, $H < 0$, friction terms become negative and gravitational waves are amplified. Such a phase of contraction is a general prediction of loop quantum cosmology (LQC) [1]. The contracting phase appears also in the string theory based descriptions of the Universe [2,3] and many others. However, in the present paper we concentrate on the LQC models where contracting and expanding regimes are joined by the bounce phase [4]. In the last years perturbations during the bounce phase were studied extensively. A recent review on this issue can be found in [5]. In particular, perturbations in the quintom bounce were studied in [6,7].

In LQC many physical results can be traced in the semiclassical approximation. In particular, dynamics of the Universe can be recovered from the quantum-corrected Friedmann equation [8,9]. A similar approach can also be applied to describe quantum gravity effects on perturbations [10], in particular to gravitational waves. It is, however, worth stressing that such an approach is rather heuristic and results obtained have to be verified by purely quantum considerations. In particular, it has not been proved yet whether the bounce phase is generally realized for the inhomogeneous loop cosmologies. However, some recent studies show that, in the case of loop quantized inhomogeneous Gowdy spacetime, the singularity is avoided [11]. In our approach inhomogeneities are treated perturbatively, and we neglect their backreaction on the background dynamics. However, in the more detailed studies these effects also have to be taken into account. In the semiclassical approach applied here, quantum gravity effects are introduced by the corrections to the classical equations of motion. For tensor modes in LQC, these effects were preliminarily studied in Ref. [12,13]. Later, an improved approach was developed [14], introducing holonomy corrections. Results of that paper are the backbone of our investigations. In this paper we assume that these corrections are valid during the whole evolution. Some preliminary studies of the influence of the holonomy corrections on gravitational wave production have been done [15–17]. However, in those papers the effects of the corrections to the source term were neglected. While in the classical approach this term vanishes (within the linear regime), in the quantum regime it does contribute. In the present paper we improve these studies by including a source term.

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Besides the holonomy corrections inverse volume corrections are also predicted in the framework of LQC. Effects of inverse volume corrections on gravitational waves were recently studied in Refs. [18,19]. However, in the flat FRW background, inverse volume corrections exhibit fiducial cell dependence. This makes those effects harder to interpret. However, in curved backgrounds this problem disappears. Since holonomy and inverse volume effects differ qualitatively, they should be studied separately. In this paper we follow this line of reasoning. We consider a consistent model where holonomy corrections influence both background and perturbation parts.

The organization of the text is the following. In Sec. II we introduce the equation for tensor modes with holonomy corrections. Then in Sec. III we define background dynamics. We consider both the model with a free scalar field and the one with multifluid potential. Subsequently, in Secs. IV and V we investigate analytically and numerically the evolution of the tensor modes. Effects of holonomy corrections are investigated. With the use of numerical computations, we calculate power spectra of the gravitational waves. In Sec. VI we summarize the results. Finally, in the Appendix we introduce gravitational waves in LQC framework, derive particular forms of the holonomy corrections, and explain the employed notation.

II. GRAVITATIONAL WAVES WITH HOLONOMY CORRECTIONS

The equation for tensor modes with LQC holonomy corrections derived in [14] is given by

$$\frac{d^2}{d\eta^2}h^i_a + 2\bar{k}\frac{d}{d\eta}h^i_a - \nabla^2 h^i_a + T_Q h^i_a = 16\pi G \Pi^i_{Qa}, \quad (1)$$

where

$$T_Q = -2\left(\frac{\bar{p}}{\bar{\mu}}\frac{\partial\bar{\mu}}{\partial\bar{p}}\right)\bar{\mu}^2\gamma^2\left(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma}\right)^4, \quad (2)$$

$$\Pi^i_{Qa} = \left[\frac{1}{3V_0}\frac{\partial\bar{H}_{\rm m}}{\partial\bar{p}}\left(\frac{\delta E^c_j\delta^j_a\delta^i_c}{\bar{p}}\right)\cos 2\bar{\mu}\gamma\bar{k} + \frac{\delta H_m}{\delta(\delta E^a_i)}\right]. \quad (3)$$

For details and an explanation of the employed notation, see the Appendix. To derive specific forms of the functions T_Q and Π^i_{Qa}, the matter content must be defined. In this paper we consider models with a scalar field. We consider both free and self-interacting fields. In that case the matter Hamiltonian is given up to second order,

$$H_m = \bar{H}_m + \frac{1}{4}\int_\Sigma d^3\mathbf{x}\frac{\bar{N}}{\sqrt{\bar{p}}}\left(\frac{1}{2}\frac{\pi^2_\phi}{\bar{p}^3} - V(\phi)\right)\delta^i_a\delta E^a_j\delta^j_b\delta E^b_i, \quad (4)$$

where the homogeneous part is given by

$$\bar{H}_m = V_0\bar{N}\bar{p}^{3/2}\left(\frac{1}{2}\frac{\pi^2_\phi}{\bar{p}^3} + V(\phi)\right). \quad (5)$$

Here the integration is constrained to fiducial volume V_0. Further physical results do not depend of this quantity.

The energy density can now be defined as

$$\rho := \frac{1}{V_0\bar{p}^{3/2}}\frac{\partial\bar{H}_m}{\partial\bar{N}}. \quad (6)$$

When the matter content is defined, one can derive particular forms of the functions (2) and (3). Expressions for the quantum holonomy corrections simplify to

$$T_Q = \frac{8\pi G}{3}\frac{\bar{p}\rho^2}{\rho_c}, \quad (7)$$

$$\Pi^i_{Qa} = \Pi_Q h^i_a = \frac{1}{2}\bar{p}\frac{\rho}{\rho_c}(2V - \rho)h^i_a. \quad (8)$$

These expressions were first derived in Ref. [18]. However, we have found a discrepancy between the expression for Π^i_{Qa} derived here and the one found in Ref. [18]. To confirm the result presented here, we show intermediate steps of the derivation in the Appendix. The difference is factor of $1/3$ inside the brackets. To derive these corrections we have applied the so-called $\bar{\mu}$ scheme of quantization. Namely, we used $\bar{\mu} = \sqrt{\Delta/\bar{p}}$, where $\Delta = 2\sqrt{3}\pi\gamma l^2_{\rm Pl}$. It is well motivated to use this particular form of the function [20]. However, other choices are, in principle, also permitted. In this paper we consider only the $\bar{\mu}$ scheme, which seems to be the best motivated.

Now the equation for the tensor modes (1) simplifies to

$$\frac{d^2}{d\eta^2}h^i_a + 2\bar{k}\frac{d}{d\eta}h^i_a - \nabla^2 h^i_a + \tilde{T}_Q h^i_a = 0, \quad (9)$$

where we have defined the total holonomy correction,

$$\tilde{T}_Q = T_Q - 16\pi G\Pi_Q = 16\pi G\bar{p}\frac{\rho}{\rho_c}\left(\frac{2}{3}\rho - V\right). \quad (10)$$

Therefore, the source term correction has also been included. This is in contrast to the analysis performed in [15–17], where this influence was neglected. In the classical theory, in fact, this term vanishes in the linear order. Therefore, when fluctuations of vacuum are considered, higher order terms can be set to zero. However, since, due to quantum corrections, the source term contributes linearly, there is no reason to neglect this term. Therefore, in the present paper we take it into account.

We introduce a new common variable,

$$u = \frac{ah_\oplus}{\sqrt{16\pi G}} = \frac{ah_\otimes}{\sqrt{16\pi G}}, \quad (11)$$

where $h^1_1 = -h^2_2 = h_\oplus$, $h^1_2 = h^2_1 = h_\otimes$, and $a = \sqrt{\bar{p}}$. Then, performing the Fourier transform

$$u(\eta, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} u(\eta, \mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{12}$$

one can rewrite Eq. (9) in the form

$$\frac{d^2}{d\eta^2} u(\eta, \mathbf{k}) + [k^2 + m_{\rm eff}^2] u(\eta, \mathbf{k}) = 0, \tag{13}$$

where $k^2 = \mathbf{k} \cdot \mathbf{k}$ and

$$m_{\rm eff}^2 = \tilde{T}_Q - \frac{a''}{a}. \tag{14}$$

In this paper we aim to solve Eq. (13). However, first we must specify the background dynamics.

III. BACKGROUND DYNAMICS

Background dynamics is governed by the effective Friedmann equation

$$\left(\frac{1}{2\bar{p}} \frac{d\bar{p}}{dt}\right)^2 = \frac{\kappa}{3} \rho \left(1 - \frac{\rho}{\rho_c}\right), \tag{15}$$

where

$$\rho_c = \frac{\sqrt{3}}{16\pi^2 \gamma^3 l_{\rm Pl}^4} \tag{16}$$

is the critical energy density. This equation can be derived by combining the Hamilton equation $\dot{\bar{p}} = \{\bar{p}, \bar{H}_m + \bar{H}_G^{\rm phen}\}$ with the scalar constraint $\bar{H}_m + \bar{H}_G^{\rm phen} = 0$.

The evolution of the scalar field component is governed by the Hamilton equations

$$\dot{\phi} = \{\phi, \bar{H}_m\} = \bar{p}^{-3/2} \pi_\phi, \tag{17}$$

$$\dot{\pi}_\phi = \{\pi_\phi, \bar{H}_m\} = -\bar{p}^{3/2} \frac{dV}{d\phi}. \tag{18}$$

The energy density and pressure of the homogeneous scalar field are expressed as follows:

$$\rho_\phi = \tfrac{1}{2}\dot{\phi}^2 + V(\phi), \tag{19}$$

$$p_\phi = \tfrac{1}{2}\dot{\phi}^2 - V(\phi). \tag{20}$$

A. Free scalar field

The energy density of the free scalar field has the form

$$\rho = \frac{1}{2} \frac{\pi_\phi^2}{\bar{p}^3}, \tag{21}$$

and the solution of the effective Friedmann equation (effective background equation) is the following:

$$\bar{p}(t) = (A + Bt^2)^{1/3}, \tag{22}$$

where

$$A = \tfrac{1}{6}\kappa \pi_\phi^2 \gamma^2 \Delta, \qquad B = \tfrac{3}{2}\kappa \pi_\phi^2. \tag{23}$$

Solution (22) represents nonsingular bouncing evolution and is discussed in Ref. [9].

For further applications, it will be useful to relate the coordinate time with the conformal one, $d\eta = dt/a(t)$. Assuming that $\eta(t = 0) = 0$, we obtain

$$\eta(t) = \frac{t}{A^{1/6}} {}_2F_1\left[\frac{1}{2}, \frac{1}{6}, \frac{3}{2}; -\frac{B}{A} t^2\right]. \tag{24}$$

B. Scalar field with a multifluid potential

One can see [21] that the restriction $p_\phi = w\rho_\phi$, where $w =$ const in the framework of effective LQC, leads to the potential in the form

$$V(\phi) = \frac{1}{2} \rho_c (1 - w) \frac{1}{\cosh^2[\sqrt{6\pi G(1 + w)}\phi]}. \tag{25}$$

The solution of the equations of motion with this potential has the simple analytic form

$$\bar{p}(t) = \bar{p}_c (1 + 6\pi G \rho_c (1 + w)^2 t^2)^{2/3(1+w)}. \tag{26}$$

It is worth mentioning that for $w = 1$ and taking

$$\bar{p}_c^{\,3} = A = \tfrac{1}{6}\kappa \pi_\phi^2 \gamma^2 \Delta \tag{27}$$

we recover the solution (22).

In analogy to the free field case, we derive

$$\eta(t) = \frac{t}{\sqrt{\bar{p}_c}} {}_2F_1\left[\frac{1}{2}, \frac{1}{3(1 + w)}, \frac{3}{2}; -6\pi G \rho_c (1 + w)^2 t^2\right]. \tag{28}$$

IV. ANALYTICAL CONSIDERATIONS

The theory of cosmological creation of particles is based on the idea of "freezing" the vacuum fluctuations. On the mathematical level, this process can be seen as a squeezing and displacement of the vacuum state $|0\rangle$. This is equivalent to the creation of particles. For noninteracting field theories the wave function is a product of the functions for the particular modes. Therefore, the degree of squeezing and coherence can be different for the particular modes and is determined by the cosmological evolution. The typical scale for which squeezing and displacement of the vacuum become important is the Hubble scale. Modes of quantum fluctuations become classical (are described as the coherent states) when crossing the Hubble radius.

To describe the process of particle creation quantitatively, one can consider Bogolyubov transformation between initial and final states. Then, computing the so-called Bogolyubov coefficients, the number of produced particles can be obtained. However, on the superhorizontal scales, one can, in principle, obtain $\omega_k^2 = k^2 + m_{\rm eff}^2 < 0$, and the interpretation in terms of particles fails. Then the quantum state cannot be interpreted in terms of particles.

Therefore, and for other reasons, it is useful to consider a correlation function which is a well-defined quantity for all energy scales. The correlation function for the tensor modes takes the form

$$\langle 0|\hat{h}^a_b(\mathbf{x},\eta)\hat{h}^b_a(\mathbf{y},\eta)|0\rangle = 4\frac{16\pi G}{a^2}\int \frac{d^3\mathbf{k}}{(2\pi)^3}|u(k,\eta)|^2 e^{-i\mathbf{k}\cdot\mathbf{r}} = \int \frac{dk}{k}\mathcal{P}_T(k,\eta)\frac{\sin kr}{kr}, \tag{29}$$

where we have defined the power spectrum

$$\mathcal{P}_T(k,\eta) = \frac{64\pi G}{a^2}\frac{k^3}{2\pi^2}|u(k,\eta)|^2. \tag{30}$$

The power spectrum can later be related to the amplitude of the CMB fluctuations. Therefore, it is crucial to determine this function.

Another way to describe the physical properties of the quantum state is by the method of Bogolyubov coefficients mentioned above. The relation between annihilation and creation operators for the initial and final states is given by the Bogolyubov transformation

$$\hat{b}_{\mathbf{k}} = B_+(k)\hat{a}_{\mathbf{k}} + B_-(k)^*\hat{a}^\dagger_{-\mathbf{k}}, \tag{31}$$

$$\hat{b}^\dagger_{\mathbf{k}} = B_+(k)^*\hat{a}^\dagger_{\mathbf{k}} + B_-(k)\hat{a}_{-\mathbf{k}}, \tag{32}$$

where $|B_+|^2 - |B_-|^2 = 1$. Since we are working in the Heisenberg description, the vacuum state does not change during the evolution. The result is that $\hat{b}_{\mathbf{k}}|0_{\rm in}\rangle = B_-(k)^*\hat{a}^\dagger_{-\mathbf{k}}|0_{\rm in}\rangle$ is different from zero when $B_-(k)^*$ is a nonzero function. This means that in the final state the graviton field considered is no longer in the vacuum state without particles. The number of produced particles in the final state is given by

$$n_{\mathbf{k}} = \tfrac{1}{2}\langle 0_{\rm in}|[\hat{b}^\dagger_{\mathbf{k}}\hat{b}_{\mathbf{k}} + \hat{b}^\dagger_{-\mathbf{k}}\hat{b}_{-\mathbf{k}}]|0_{\rm in}\rangle = |B_-(k)|^2. \tag{33}$$

The energy density of gravitons is given by

$$d\rho_{\rm gw} = 2\cdot\hbar\omega\cdot\frac{4\pi\omega^2 d\omega}{(2\pi c)^3}\cdot|B_-(k)|^2, \tag{34}$$

where we used definition (33). To describe the spectrum of gravitons, it is common to use the parameter

$$\Omega_{\rm gw}(\nu) = \frac{\nu}{\rho_*}\frac{d\rho_{\rm gw}}{d\nu}, \tag{35}$$

where $\rho_{\rm gw}$ is the energy density of gravitational waves and ρ_* is the present critical energy density.

A. Free scalar field

Based on solution (22) we derive

$$m^2_{\rm eff} = \frac{\kappa^2\pi^4_\phi}{4}\frac{(t^2 + \frac{1}{9}\gamma^2\Delta)}{(A+Bt^2)^{5/3}} \geq 0. \tag{36}$$

We show this function in Fig. 1. We compare it with the classical expression

$$m^2_{\rm eff}(\tilde{T}_Q = 0) = \frac{\kappa^2\pi^4_\phi}{4}\frac{(t^2 - \frac{1}{3}\gamma^2\Delta)}{(A+Bt^2)^{5/3}}. \tag{37}$$

The difference is significant since now the effective mass is a non-negative function, $m^2_{\rm eff} \geq 0$. One can also compare this to the case when the source term corrections were neglected. Then, as can be found in Ref. [16], the effective mass is negative in the central region, similar to the classical case. Here the difference is crucial and has important consequences. Namely, since $m^2_{\rm eff} \geq 0$, we have $\omega^2_k \geq 0$ and the interpretation in terms of particles is possible on all scales. This also becomes a nice property when the Hamiltonian of the perturbations is minimized to find a proper vacuum state. It can be shown that, when $\omega^2_k \geq 0$, the Hamiltonian has a minimum for all k, and a well-defined vacuum can be found. Otherwise, for some $k < k_{\rm x}$, the lowest energy instantaneous vacuum state does not exist.

Now we are going to consider the prebounce limit. Taking $|t| \to \infty$, we find

$$m^2_{\rm eff} \to \frac{1}{4}\frac{1}{\eta^2}. \tag{38}$$

The normalized solution of Eq. (13) has the form

$$u(k,\eta) = \sqrt{\frac{\pi}{2}}e^{i\pi/4}\frac{1}{\sqrt{2k}}\sqrt{-\eta k}H^{(1)}_0(-\eta k). \tag{39}$$

We have chosen here the advanced modes and performed normalization with the use of the Wronskian condition. In the superhorizontal limit $-\eta k \ll 1$, we can apply the approximation

$$H^{(1)}_0(x) \simeq 1 + i\frac{2}{\pi}\left[\ln\left(\frac{x}{2}\right) + \gamma_E\right], \tag{40}$$

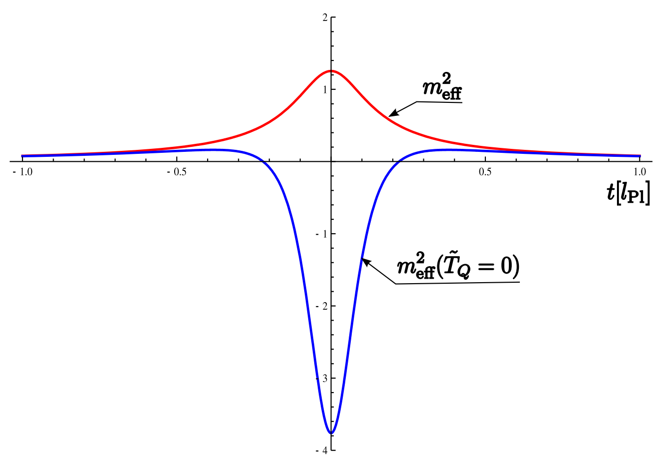

FIG. 1 (color online). Evolution of the effective masses $m^2_{\rm eff}$ and $m^2_{\rm eff}(\tilde{T}_Q = 0)$. In this figure we have assumed $\pi_\phi = 0.1 l_{\rm Pl}$.

where $\gamma_E = 0.577\,21\ldots$ is the Euler-Mascheroni constant. The expression for the power spectrum in the superhorizontal limit is therefore

$$\mathcal{P}_T(k) = \mathcal{A}k^3\left\{1 + \frac{4}{\pi^2}\left[\ln\left(-\frac{k\eta}{2}\right) + \gamma_E\right]^2\right\}, \tag{41}$$

where

$$\mathcal{A} = 4\sqrt{\frac{2}{\pi}}\left(\frac{3}{2}\right)^{1/6}\left(\frac{l_{\rm Pl}}{\pi_\phi}\right). \tag{42}$$

To investigate k dependence in formula (41), we define the spectral index

$$n_T = \frac{d\ln\mathcal{P}_T(k)}{d\ln k} \tag{43}$$

and obtain

$$n_T = 3 + \frac{8}{\pi^2}\frac{\ln(-\frac{k\eta}{2}) + \gamma_E}{1 + \frac{4}{\pi^2}[\ln(-\frac{k\eta}{2}) + \gamma_E]^2}. \tag{44}$$

We show this function for some fixed time in Fig. 2. We find that the resulting spectral index is blue and approaches $n_T = 3$ for $k \to 0$. This blue-tilted spectrum was predicted earlier in [16]. Recent investigations suggest that for inflationary cosmology with holonomy corrections the spectrum obtained is also blue-tilted and $n_T = 3$ at superhorizontal scales [17].

B. Multifluid potential

Now we are going to perform a similar analysis for the model with a multifluid potential. We obtain the formula

$$m^2_{\rm eff} = \bar{p}_c\frac{1}{8}\kappa^2\rho_c^2(1+w)^2(3w-1)(1+6\pi G\rho_c(1+w)^2t^2)^\alpha \times\left\{t^2 - \frac{4}{3}\frac{\Delta\gamma^2}{(1+w)(3w-1)}\left[1 - \frac{2}{3}\frac{(1+3w)}{(1+w)}\right]\right\}, \tag{45}$$

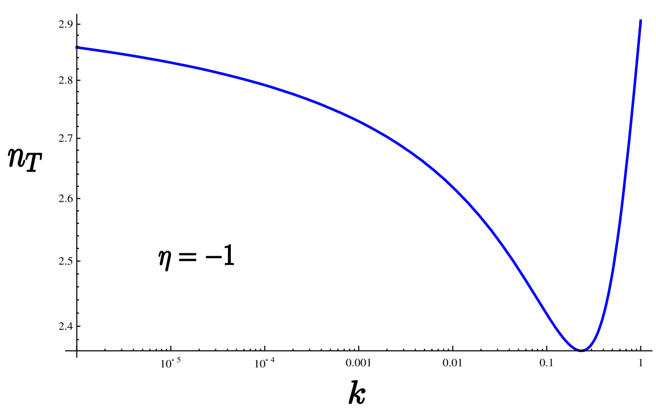

FIG. 2 (color online). Running spectral index on superhorizontal scales.

where

$$\alpha = -\frac{2}{3}\frac{2+3w}{1+w}. \tag{46}$$

In the limit $|t| \to \infty$ we obtain

$$m^2_{\rm eff} \to \frac{6w-2}{(1+3w)^2}\frac{1}{\eta^2}, \tag{47}$$

where we changed the time to conformal. An advanced, normalized solution of Eq. (13) in the considered limit is

$$u(k,\eta) = \sqrt{-k\eta}\sqrt{\frac{\pi}{4k}}e^{i(\pi/2)(|\nu|+(1/2))}H^{(1)}_{|\nu|}(-\eta k), \tag{48}$$

where

$$\nu^2 = \frac{9}{4}\frac{(1-w)^2}{(1+3w)^2}. \tag{49}$$

The power spectrum of the perturbations is then given as

$$\mathcal{P}_T(k) \propto (-k\eta)^{3-2|\nu|}, \tag{50}$$

where the superhorizontal approximation

$$H^{(1)}_n(x) \simeq -\frac{i}{\pi}\Gamma(n)\left(\frac{x}{2}\right)^{-n} \quad \text{for } x \ll 1 \tag{51}$$

has been used. It must be stressed that the above formula does not hold for $n = 0$ ($w = 1$). In that case another expansion (40) must be applied.

It is worth mentioning that the scale invariant spectrum $|\nu| = \frac{3}{2}$ is recovered both for $w = -1$ and $w = 0$, as it can be directly seen from (49). This duality was investigated in Ref. [22] in the context of the free scalar field perturbations.

C. Subhorizontal solutions

So far we have only been concerned with the prebounce phase. Now we are going to evolve modes through the bounce. We first consider the modes which stay under the Hubble radius before the bounce. For those modes the initial vacuum state is given by

$$u_{\rm in} = \frac{e^{-ik\eta}}{\sqrt{2k}}. \tag{52}$$

This can be obtained as a limit of the mode function (39) for $-k\eta \gg 1$. To be specific, let us consider the model with a free scalar field and $\pi_\phi = 0.1 l_{\rm Pl}$.

At the Hubble radius we have

$$k_{\rm H} = a|H|, \tag{53}$$

which is shown in Fig. 3. We see that for initial time, let us say $t = -1000 l_{\rm Pl}$, all modes with $k > 0.003$ are well described by the function (52). These solutions, however, do not hold during the bounce phase. Close to the bounce one can approximate

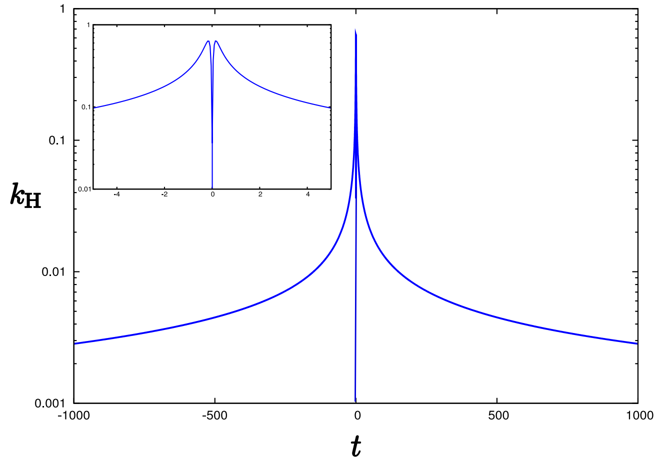

FIG. 3 (color online). Evolution of the Hubble wave number $k_H = a|H|$. Here $\pi_\phi = 0.1 l_{Pl}$.

$$m_{eff}^2 \approx m_{eff}^2(t=0) = \frac{1}{(54)^{1/3}} \kappa (\pi_\phi \rho_c)^{2/3} \equiv k_0^2. \tag{54}$$

For the considered conditions we obtain $k_0 \simeq 1.12$. In this approximation the solutions during the bounce phase are

$$u_{bounce} = \frac{A_k}{\sqrt{2\Omega}} e^{-i\Omega\eta} + \frac{B_k}{\sqrt{2\Omega}} e^{i\Omega\eta}, \tag{55}$$

where $\Omega = \sqrt{k^2 + k_0^2}$. Finally, in the post-bounce phase we have a superposition of advanced and retarded modes,

$$u_{out} = \frac{\alpha_k}{\sqrt{2k}} e^{-ik\eta} + \frac{\beta_k}{\sqrt{2k}} e^{ik\eta}. \tag{56}$$

Here the relation $|\alpha_k|^2 - |\beta_k|^2 = 1$ holds, as a consequence of the normalization condition. Now we have to match the solutions from the three considered regions to determine the coefficients α_k and β_k. In order to do that we must specify a time when the matching is performed. We fix this time in the mirror points $-t_- = t_+$, where H^2 reaches its maximal value. Then $-t_- = t_+ = t_0$, where

$$t_0 = \frac{1}{\sqrt{24\pi G\rho_c}}. \tag{57}$$

With the use of Eq. (24) we obtain

$$\eta_0 = \eta(t_0) = \frac{{}_2F_1[\frac{1}{2}, \frac{1}{6}, \frac{3}{2}; -1]}{\sqrt{3\kappa}\rho_c^{1/3}(\pi_\phi^2/2)^{1/6}}. \tag{58}$$

For the considered setup we obtain $\eta_0 \simeq 0.285$.

In order to derive formulas for the coefficients α_k and β_k, we define the matrices

$$\mathbf{M}_0 = \begin{pmatrix} \frac{e^{-ik\eta_-}}{\sqrt{2k}} & \frac{e^{ik\eta_-}}{\sqrt{2k}} \\ -i\sqrt{\frac{k}{2}}e^{-ik\eta_-} & i\sqrt{\frac{k}{2}}e^{ik\eta_-} \end{pmatrix}, \tag{59}$$

$$\mathbf{M}_1 = \begin{pmatrix} \frac{e^{-i\Omega\eta_-}}{\sqrt{2\Omega}} & \frac{e^{i\Omega\eta_-}}{\sqrt{2\Omega}} \\ -i\sqrt{\frac{\Omega}{2}}e^{-i\Omega\eta_-} & i\sqrt{\frac{\Omega}{2}}e^{i\Omega\eta_-} \end{pmatrix}, \tag{60}$$

$$\mathbf{M}_2 = \begin{pmatrix} \frac{e^{-i\Omega\eta_+}}{\sqrt{2\Omega}} & \frac{e^{i\Omega\eta_+}}{\sqrt{2\Omega}} \\ -i\sqrt{\frac{\Omega}{2}}e^{-i\Omega\eta_+} & i\sqrt{\frac{\Omega}{2}}e^{i\Omega\eta_+} \end{pmatrix}, \tag{61}$$

$$\mathbf{M}_3 = \begin{pmatrix} \frac{e^{-ik\eta_+}}{\sqrt{2k}} & \frac{e^{ik\eta_+}}{\sqrt{2k}} \\ -i\sqrt{\frac{k}{2}}e^{-ik\eta_+} & i\sqrt{\frac{k}{2}}e^{ik\eta_+} \end{pmatrix}. \tag{62}$$

Then the matching conditions can be economically written as

$$\begin{pmatrix} \alpha_k \\ \beta_k \end{pmatrix} = \mathbf{M}_3{}^{-1}\mathbf{M}_2\mathbf{M}_1{}^{-1}\mathbf{M}_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{63}$$

Multiplying these matrices we obtain

$$\alpha_k = \frac{-i\cos(2\eta_0 k) + \sin(2\eta_0 k)}{2k\Omega}[2ik\Omega\cos(2\eta_0\Omega) + (k^2 + \Omega^2)\sin(2\eta_0\Omega)], \tag{64}$$

$$\beta_k = -\frac{i(k^2 - \Omega^2)\sin(2\eta_0\Omega)}{2k\Omega}. \tag{65}$$

The resulting square of the amplitude for the out-state modes is

$$|u_{out}|^2 = \frac{(k^2+\Omega^2)^2 - k_0^4\cos[4\eta_0\Omega] - k_0^2\sin[2\eta_0\Omega]((k+\Omega)^2\sin[2k(\eta-\eta_0)+2\Omega\eta_0] - (k-\Omega)^2\sin[2k(\eta-\eta_0)-2\Omega\eta_0])}{8k^3(k^2+k_0^2)}. \tag{66}$$

Based on this result one can calculate the power spectrum of perturbations. We show this spectrum in Fig. 4. The obtained spectrum exhibits subhorizontal oscillations. Applying analogy with the Schrödinger equation, this effect can be intuitively understood. Namely, the mode equations are equivalent to a one-dimensional Schrödinger equation with potential $V = -m_{eff}^2$. Here the spatial variable is replaced by the conformal time η. In the employed approximation the potential is a square well of width $2\eta_0$ and depth $m_{eff}^2(t=0)$. Therefore the evolution of the given mode can be seen as a transition of a particle over the potential well. Amplifications of the amplitude of the transmission correspond to resonances between the width of the potential and the phase shift.

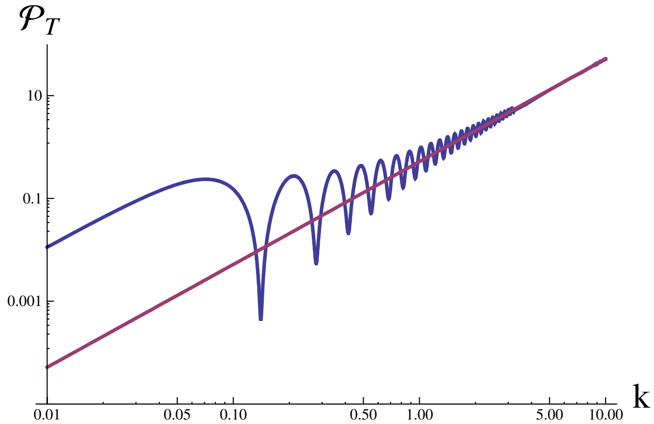

FIG. 4 (color online). Oscillating tensor power spectrum of the subhorizontal modes at $t = 50 l_{\rm Pl}$. The straight line represents the reference spectrum, $\mathcal{P}_T \propto k^2$.

It can be shown that the obtained coefficients (α_k, β_k) are in fact the Bogolyubov coefficients $\alpha_k = B_+$ and $\beta_k = B_-$. Therefore the number of produced gravitons is given by

$$n_k = |\beta_k|^2 = \frac{k_0^4 \sin^2(2\eta_0\sqrt{k^2 + k_0^2})}{k^2(k_0^2 + k^2)}. \tag{67}$$

We show this dependence in Fig. 5. Now it is straightforward to calculate the parameter $\Omega_{\rm gw}$. We show this function in Fig. 6. We compare it with the obtained low energy approximation $\Omega_{\rm gw} \propto \nu^{-2}$. The obtained values of $\Omega_{\rm gw}$ are many orders of magnitude below the present threshold for detection. The results obtained were performed for some simplified model and for the fixed value of π_ϕ. In particular, in the low energy limit $\Omega_{\rm gw} \sim \pi_\phi^{2/3}$; therefore the effect of varying π_ϕ is considerable. However, we do not expect significant changes due to the approximations performed. This statement will be confirmed by the numerical simulations in Sec. V.

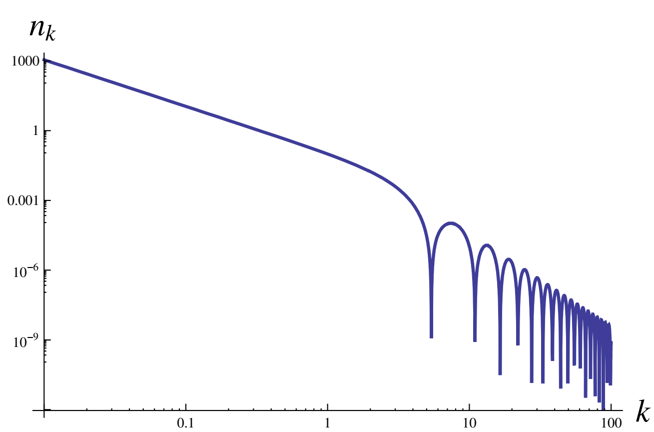

FIG. 5 (color online). Occupation number of the gravitons in the post-bounce state.

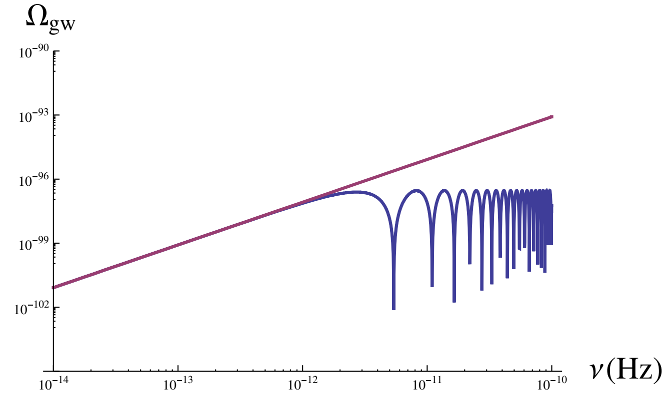

FIG. 6 (color online). Parameter $\Omega_{\rm gw}$ (bottom curve). The straight line represents the low energy approximation $\Omega_{\rm gw} \propto \nu^{-2}$.

D. Superhorizontal solutions

In the previous subsection we have shown solutions of the mode equation (13) in the subhorizontal limit. Now we are going to study the superhorizontal $k \rightarrow 0$ limit.

We introduce new variable in the form

$$f = \sqrt{a} u, \tag{68}$$

and change the conformal time to the coordinate one, $dt = a d\eta$. Then Eq. (13) can be rewritten in the form

$$\frac{d^2 f}{dt^2} + \Omega^2(k, t) f = 0. \tag{69}$$

Here the parameter $\Omega^2(k, t)$ is defined as follows:

$$\Omega^2(k, t) = \left(\frac{k}{a}\right)^2 + \epsilon \frac{\tilde{T}_Q}{a^2} - \frac{3}{2}\left(\frac{\ddot{a}}{a} + \frac{1}{2} H^2\right). \tag{70}$$

We have introduced here the parameter ϵ to trace the effects of the holonomy corrections in the later equations. In the classical limit we should take $\epsilon = 0$, while in the presence of the holonomy corrections, $\epsilon = 1$. Taking $k = 0$ and introducing a new complex variable $z \in \mathbb{C}$,

$$z = \tfrac{1}{2} + i\tfrac{1}{2}\sqrt{6\pi G \rho_c}(1 + w)t, \tag{71}$$

we can rewrite Eq. (69) in the following form:

$$\frac{d^2 f}{dz^2} + Q(z) f = 0, \tag{72}$$

where

$$Q(z) = \frac{\alpha_2 z^2 + \alpha_1 z + \alpha_0}{z^2(z - 1)^2}. \tag{73}$$

The coefficients are

$$\alpha_0 = \frac{9(1 + 2w) - 4\epsilon(1 + 3w)}{36(1 + w)^2}, \tag{74}$$

$$\alpha_1 = -\frac{w}{(1+w)^2}, \tag{75}$$

$$\alpha_2 = \frac{w}{(1+w)^2}, \tag{76}$$

and it will be useful later to remember that $\alpha_1 + \alpha_2 = 0$. Now introducing the new variable

$$f(z) = z^L(z-1)^K g(z) \tag{77}$$

with

$$L = \frac{c}{2} \tag{78}$$

$$K = \frac{a+b+1-c}{2}, \tag{79}$$

one can rewrite Eq. (72) as a hypergeometric equation,

$$z(1-z)\frac{d^2g}{dz^2} + [c-(a+b+1)z]\frac{dg}{dz} - abg = 0. \tag{80}$$

The solution of this equation is given by the hypergeometric functions

$$g(z) = C_2F_1(a, b, c; z). \tag{81}$$

Furthermore, we have a system of equations for the coefficients,

$$\alpha_0 + L(L-1) = 0, \tag{82}$$

$$\alpha_1 + ab - 2KL - 2L(L-1) = 0, \tag{83}$$

$$\alpha_2 - ab + 2KL + L(L-1) + K(K-1) = 0. \tag{84}$$

One can see that, since $\alpha_1 + \alpha_2 = 0$, we have either $K = L$ or $K = 1 - L$, where $L = \frac{1}{2}(1 \pm \sqrt{1-4\alpha_0})$. For $K = L$ we find

$$a = \tfrac{1}{2}(2c - 1 \pm \sqrt{1+4\alpha_1}), \tag{85}$$

$$b = 2c - a - 1, \tag{86}$$

$$c = 2L = 1 \pm \sqrt{1-4\alpha_0}, \tag{87}$$

and while $K = 1 - L$ we have

$$a = \tfrac{1}{2}(1 \pm \sqrt{1+4\alpha_1}), \tag{88}$$

$$b = 1 - a, \tag{89}$$

$$c = 2L = 1 \pm \sqrt{1-4\alpha_0}. \tag{90}$$

As an exemplary solution we consider the $w = 1$ case, both classically ($\epsilon = 0$) and with holonomy corrections to the mode equation ($\epsilon = 1$). Then, since $\alpha_1 = -\alpha_2 = -1/4$ we have

$$a = b = c - \tfrac{1}{2}, \tag{91}$$

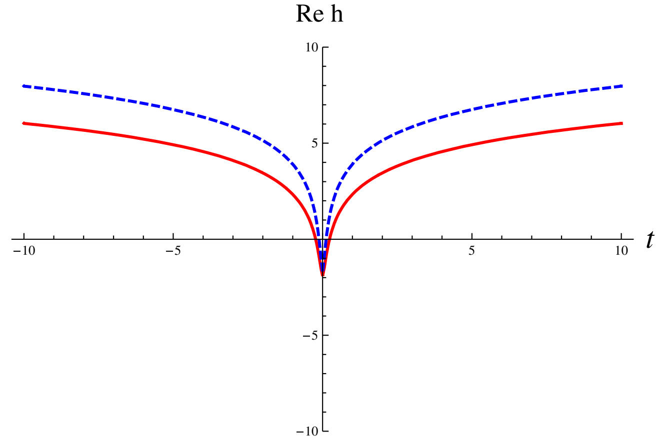

FIG. 7 (color online). Real components of the h variable. The dashed (blue) line represents the solution with $c_+(\epsilon = 1)$ while the solid (red) line represents the solution with $c_+(\epsilon = 0)$.

where

$$c_\pm(\epsilon = 1) = 1 \pm \frac{\sqrt{47}}{6} \quad \text{and} \quad c_\pm(\epsilon = 0) = 1 \pm \frac{\sqrt{7}}{2}. \tag{92}$$

In Fig. 7 we show solutions for the real components of the h variable. In Fig. 8 we show solutions for the imaginary components of the h variable. In Fig. 9 we show solutions for the absolute value of the h variable.

As it can be seen, solutions with and without the quantum holonomy corrections are qualitatively similar. Another observation is that for times $t \gg 1$, evolution takes a logarithmic form. This result can be understood by considering Eq. (13) in the classical limit and taking $k \to 0$. Then one can find the approximate solution in the form

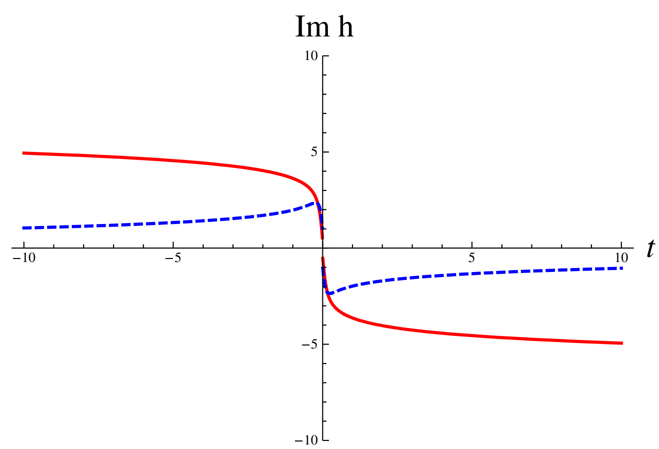

FIG. 8 (color online). Imaginary components of the h variable. The dashed (blue) line represents the solution with $c_+(\epsilon = 1)$ while the solid (red) line represents the solution with $c_+(\epsilon = 0)$.

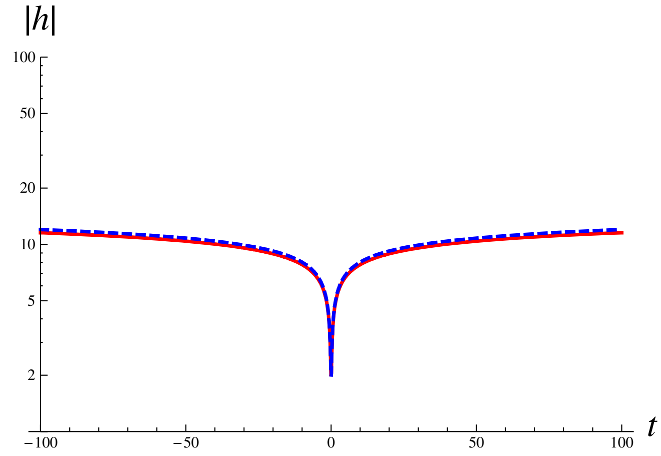

FIG. 9 (color online). Absolute values of the h variable. The dashed (blue) line represents the solution with $c_+(\epsilon = 1)$ while the solid (red) line represents the solution with $c_+(\epsilon = 0)$.

$$h \simeq A_k + B_k \int^{\eta} \frac{d\eta'}{a^2(\eta')}, \tag{93}$$

where A_k and B_k are some constants. For the models considered in the present paper, we have $a \propto (\pm\eta)^{2/(1+3w)}$ and $\pm\eta \propto (\pm t)^{(1+3w)/3(1+w)}$. Here we have a $+$ for the expanding phase and a $-$ for the contracting one. Therefore, for the considered $w = 1$ case we find

$$h \simeq \tilde{A}_k + \tilde{B}_k \ln(\pm t) \quad \text{for } |t| \gg 1, \tag{94}$$

in agreement with the solutions found in this subsection.

V. NUMERICAL INVESTIGATIONS

As it was shown in the previous section, analytic solutions of the mode equation are available only in certain limits, namely, for both $t^2 \to \infty$ and $k \to 0$. Also, for $k \to \infty$ an approximate solution was found. It is, however, not sufficient to describe the whole spectrum of the gravitational waves produced in the bounce phase since the interesting intermediate regimes are unexplored. Therefore a numerical analysis is required.

In the numerical computations we are going to solve the autonomous system of equations

$$\frac{du}{d\eta} = \pi_u, \tag{95}$$

$$\frac{d\pi_u}{d\eta} = -[k^2 + m^2_{\text{eff}}(t)]u, \tag{96}$$

$$\frac{dt}{d\eta} = a(t), \tag{97}$$

where $a(t)$ and $m^2_{\text{eff}}(t)$ are defined for particular background dynamics. In the considered models with a free scalar field and a multifluid potential, these functions are given by analytical expressions. Since canonical variables $u, \pi_u \in \mathbb{C}$, we decompose them as follows:

$$u = u_1 + iu_2, \tag{98}$$

$$\pi_u = \pi_{u1} + i\pi_{u2}. \tag{99}$$

Now it is crucial to define proper initial conditions for $(u_1, u_2, \pi_{u1}, \pi_{u2})$ for some time η_0. It is ambiguous how to choose a proper vacuum defined on the cosmological background. However, on subhorizontal scales, when the Minkowski space approximation holds, we can set

$$u_1(\eta_0) = \frac{1}{\sqrt{2k}}\cos(k\eta_0), \tag{100}$$

$$u_2(\eta_0) = -\frac{1}{\sqrt{2k}}\sin(k\eta_0) \tag{101}$$

and

$$\pi_{u1}(\eta_0) = -\sqrt{\frac{k}{2}}\sin(k\eta_0), \tag{102}$$

$$\pi_{u2}(\eta_0) = -\sqrt{\frac{k}{2}}\cos(k\eta_0) \tag{103}$$

at some time η_0. Here we set initial values like in the model of subhorizontal modes studied in the previous section. Therefore the analysis is correct for the modes with $k > 0.003$.

In Fig. 10 we plot the evolution of the $k = 0.1$ mode during the bounce phase. We compare here the evolution of modes with and without holonomy corrections to the mode equations. We see that close to the turning point the effects of the holonomy corrections become significant. However,

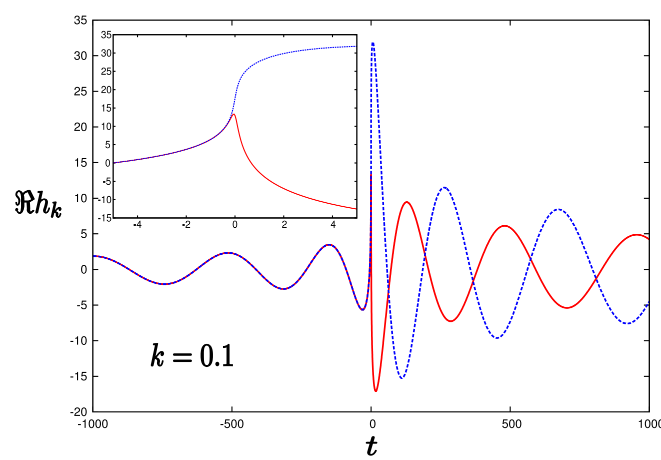

FIG. 10 (color online). Evolution of the modes with $k = 0.1$. The dotted (blue) curve represents the solution of the mode equations with holonomy corrections. The solid (red) curve represents the solution of the mode equations without holonomy effects.

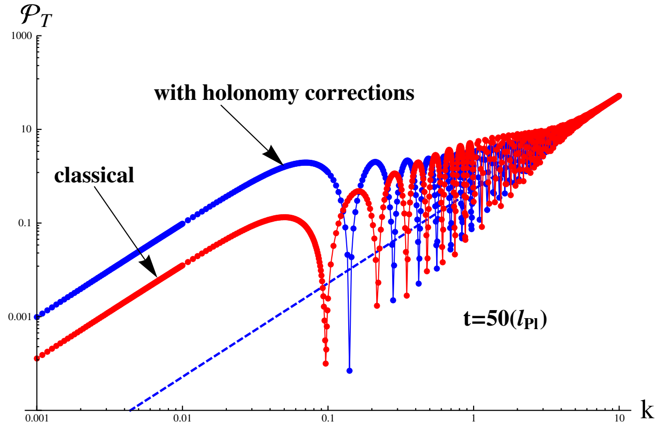

FIG. 11 (color online). Comparison between subhorizontal spectra obtained with and without holonomy corrections to the mode equation. The dashed (blue) line represents the rescaled initial vacuum power spectrum.

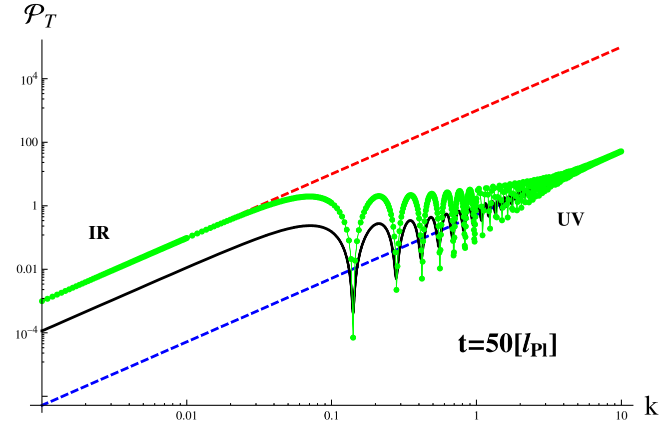

FIG. 12 (color online). Whole post-bounce tensor spectra in the presence of the holonomy corrections. Green points are from the numerical simulations. The black line is the analytical spectrum from the model given by Eq. (66). The dashed (red and blue) lines represent UV and IR behaviors; in both cases $\mathcal{P}_T \propto k^2$.

the further oscillating evolution does not change qualitatively. The difference is that there is some suppression of the amplitude of perturbations due to the quantum corrections. This feature can also be seen in Fig. 11, where classical- and quantum-corrected tensor power spectra are shown. We find, comparing with the classical case, that quantum holonomy effects amplify low energy modes. Therefore, tensor power spectra increase by about 1 order of magnitude. For high energies classical spectra start to dominate slightly. It is also worth noticing that oscillations do not overlap.

To impose initial conditions on the superhorizontal scales, one can use an instantaneous vacuum. This is, however, possible only for values of k fulfilling $\omega_k^2 \geq 0$. As we found earlier, this condition is fulfilled for all k in the model with a free scalar field. Therefore the initial instantaneous vacuum state can be defined on all length scales. It can be shown that the Hamiltonian of perturbations at time η_0 is minimized for

$$u(\eta_0) = \frac{1}{\sqrt{2\omega_k}}, \tag{104}$$

$$\pi_u(\eta_0) = -i\sqrt{\frac{\omega_k}{2}}, \tag{105}$$

where $\omega_k = \sqrt{k^2 + m_{\text{eff}}^2}$.

In Fig. 12 we show the tensor power spectrum at the post-bounce stage ($t = 50 l_{\text{Pl}}$) with instantaneous vacuum initial conditions imposed at $t = -1000 l_{\text{Pl}}$. The characteristic features of the spectrum are oscillations. Moreover, both UV and IR behaviors are of the form $\mathcal{P}_T \propto k^2$. We see that the analytical model given by Eq. (66) overlaps fairly well with the numerical results. In particular, the structure of the oscillations is exactly recovered. Also, the asymptotic behaviors are consistent. The evident discrepancy is the difference in the total amplitude. In fact, this difference can be suitably adjusted by varying the parameters of the model, η_0 and k_0. Then the low energy behavior can be exactly recovered. However, this introduces an additional phase shift, and the structures of the oscillation no longer overlap. It is also important to note that the effect of the instantaneous vacuum initial conditions is negligible in the range studied. Therefore the Minkowski vacuum approximation is still valid.

The power spectrum obtained can now be applied as an initial condition for the inflationary modes. We expect the superhorizontal part of the spectrum not to change during the inflationary phase. However, the UV part becomes nearly flat (depending on the model of inflation). It is possible that the oscillating features will also survive, giving the footprints of the bouncing phase. However, further analysis has to be performed to confirm these speculations. In particular, the inflationary power spectrum with the obtained bouncing initial conditions must be calculated. Then it will be possible to compute the B-type polarization spectra of CMB. Therefore, a potential way to relate the quantum cosmological effects with the low energy physics becomes available.

VI. SUMMARY

In this paper we have considered the influence of loop quantum gravity effects on the propagation of gravitational waves in the flat FRW cosmological background. The considerations presented are based on the semiclassical approach, where quantum effects are introduced by corrections to the classical equations of motion. This approach was successfully applied to the homogeneous models. In this case good agreement between the results of fully quantum and semiclassical analyses was found. Here we have applied the semiclassical approach to the inhomogeneous model where inhomogeneity is treated perturbatively. Therefore perturbations have no influence on background. In general, both classical and quantum backreaction effects can be important close to the bounce phase. Here we assumed that these effects can be neglected. We have not considered the effects of the quantum fluctuations of background on the inhomogeneities either. Quantum effects were introduced by the so-called holonomy corrections. In the homogeneous models these corrections lead to the absence of the initial singularity and the emergence of the bounce phase. Effects of the other known type of LQG corrections, the inverse volume ones, were studied earlier in numerous papers. Here we considered a self-consistent model where holonomy effects influence both background and perturbations (gravitational waves). In the earlier studies effects on background and perturbations were studied independently. In particular, in Ref. [16] a model of gravitational wave production during the holonomy-induced bounce phase was investigated. In Refs. [15,17] effects of holonomy corrections on gravitational waves in the inflationary phase were studied. However, quantum effects on the background dynamics were neglected there. Moreover, the quantum-corrected source term was not taken into account in those studies. The linear part of this term vanishes in the classical limit. However, it contributes while holonomy corrections are present. Therefore, the source term has to be taken into account in the full treatment. In the present paper we have included effects of this term.

We have considered models with both a free scalar field and a self-interacting field with multifluid potential. In both cases the scalar field is a monotonic function and can be treated as an internal time variable.

We have shown that in the model with the free field, the effective mass term $m^2_{\rm eff}$ for gravitational waves is a nonnegative function. This is not the case for the models with multifluid potential. We have found solutions of the mode function in the prebounce phase and determined the power spectra of the perturbations obtained. Then we considered subhorizontal solutions during the bounce phase. We matched the solutions from the prebounce, bounce, and post-bounce phases. Based on this we have found the power spectrum of gravitational waves and determined the Bogolyubov coefficients. Next, the number of produced gravitons $n_{\mathbf{k}}$ and the parameter $\Omega_{\rm gw}$ were calculated. We have found that $\Omega_{\rm gw}$ reaches 10^{-96}, which is far below any observational bounds. These results were obtained for the fixed parameter $\pi_\phi = 0.1 l_{\rm Pl}$.

Based on analytical considerations we have found that the power spectrum exhibits oscillations on subhorizontal scales. An intuitive explanation of this effect was given. We have also solved the model analytically in the superhorizontal limit. These results indicate that quantum corrections do not introduce qualitative differences in the power spectrum on these scales. Therefore the lack of power obtained on large scales is a feature of the bouncing evolution and not of the quantum corrections to the mode equation.

Subsequently, we have investigated the model numerically. We have proved the presence of the oscillations that emerged from the simplified analytical considerations. Both numerical and analytical results were compared. We have found good qualitative and quantitative agreement. We have also proved the earlier observation that quantum corrections do not introduce qualitative differences in the power spectrum. The only differences observed were in the total amplitude and the phase of oscillations.

Imposing an initial instantaneous vacuum state, we have also studied the low energy part of the power spectrum. Therefore, we have finally found the full shape of the tensor power spectrum. This spectrum can be used to study further phenomenological consequences. In particular, it can be applied as an initial condition for the inflationary modes. Then we expect that the subhorizontal part of the spectrum becomes flat while the superhorizontal part survives. It is also possible that subhorizontal oscillations survive as features of the dominant, nearly flat, inflationary spectrum. Therefore, two observational effects of the bouncing phase can be distinguished: oscillations and lack of power on superhorizontal scales. These effects can potentially be tested with future CMB missions like Planck [23] or the proposed CMBPol [24]. Especially promising are observations of the CMB polarization. Here the bounce can lead to the suppression of low multipoles in the B-type spectrum. At present, projects like Clover [25], QUaD [26], and QUIET [27] are aiming to detect this spectrum, and the first results are expected in the near future. Therefore, the next step is to derive quantitative predictions of the CMB features from the model presented.

ACKNOWLEDGMENTS

The author is grateful to Francesca Vidotto for discussion during the conference "Quantum Gravity in Cracow^2" 19-21 XII 2008, Poland, where some of these results were presented.

APPENDIX: LOOP QUANTUM GRAVITY WITH GRAVITATIONAL WAVES

LQG describes the gravitational field as an $SU(2)$ non-Abelian gauge field using background-independent methods. The canonical fields are given by the Ashtekar variables ($A = A^i_a \tau_i dx^a$, $E = E^a_i \tau^i \partial_a$) which take value in $\mathfrak{su}(2)$ and $\mathfrak{su}(2)^*$ algebras, respectively, and fulfill the Poisson bracket,

$$\{A^i_a(\mathbf{x}), E^b_j(\mathbf{y})\} = \gamma\kappa \delta^b_a \delta^i_j \delta^{(3)}(\mathbf{x}-\mathbf{y}), \qquad \text{(A1)}$$

where $\kappa = 8\pi G$ and γ is the Barbero-Immirzi parameter. These variables are analogues of the vector potential and the electric field in electrodynamics. The Ashtekar variables are related to the triad representation. In LQG gauge fields describe only the spatial part Σ, while time is treated separately.

In cosmological applications we perturb basic variables around a background,

$$E^a_i = \bar{E}^a_i + \delta E^a_i, \tag{A2}$$

$$A^i_a = \bar{A}^i_a + \delta A^i_a. \tag{A3}$$

For the spatially flat FRW background, components have the following form:

$$\bar{E}^{\ a}_{\ i} = \bar{p}\delta^a_i, \tag{A4}$$

$$\bar{A}^{\ i}_{\ a} = \gamma\bar{k}\delta^i_a, \tag{A5}$$

where $\bar{p} = a^2$ and $\bar{k} = \dot{\bar{p}}/2\bar{p}$. Perturbations can be split for the scalar, vector, and tensor parts. For the purpose of this paper we consider here only the gravitational waves (tensor part). Tensor perturbations of the flat FRW metric are introduced as follows:

$$g_{00} = -N^2 + q_{ab}N^aN^b = -\bar{N} = -a^2,$$
$$g_{0a} = q_{ab}N^b = 0, \qquad g_{ab} = q_{ab} = a^2[\delta_{ab} + h_{ab}],$$

with the conditions $h^a_a = \partial_a h^a_b = 0$ and $|h_{ab}| \ll 1$. In the TT gauge $h^1_1 = -h^2_2 = h_\oplus$ and $h^1_2 = h^2_1 = h_\otimes$.

Now we are going to perturb the Hamiltonian of the theory. The full Hamiltonian is composed of the gravitational and $H_{\rm G}$ and matter $H_{\rm m}$ parts. Hamiltonian $H_{\rm G}$ takes the form of a linear combination of the constraints,

$$H_{\rm G} = \int_\Sigma d^3\mathbf{x}(N^iG_i + N^aC_a + NS),$$

a spatial diffeomorphism constraint,

$$C_a = E^b_iF^i_{ab} - (1-\gamma^2)K^i_aG_i,$$

a Gauss constraint,

$$G_i = D_aE^a_i = \partial_aE^a_i + \epsilon_{ijk}A^j_aE^a_k,$$

and a scalar constraint,

$$S = \frac{E^a_iE^b_j}{\sqrt{|\det E|}}[\varepsilon^{ij}_{\ \ k}F^k_{ab} - 2(1+\gamma^2)K^i_{[a}K^j_{b]}],$$

where $F = dA + \frac{1}{2}[A, A]$. However, thanks to the quantum gravity effect, this Hamiltonian undergoes modifications. These modifications can be introduced on the phenomenological level by the replacement

$$\bar{k} \to \frac{\sin n\bar{\mu}\gamma\bar{k}}{n\bar{\mu}\gamma} \tag{A6}$$

in the classical expressions. Here

$$\bar{\mu} = \sqrt{\frac{\Delta}{\bar{p}}} \quad \text{where } \Delta = 2\sqrt{3}\pi\gamma l^2_{\rm Pl}.$$

We call these holonomy corrections. The factor n can be fixed from the requirement of the anomaly cancellation [14,28]. The effective second order Hamiltonian with holonomy corrections takes the form

$$\begin{aligned} H^{\rm phen}_G = \frac{1}{16\pi G}\int_\Sigma d^3x\bar{N}\bigg[&-6\sqrt{\bar{p}}\left(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma}\right)^2 \\ &- \frac{1}{2\bar{p}^{3/2}}\left(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma}\right)^2(\delta E^c_j\delta E^d_k\delta^k_c\delta^j_d) \\ &+ \sqrt{\bar{p}}(\delta K^j_c\delta K^k_d\delta^c_k\delta^d_j) - \frac{2}{\sqrt{\bar{p}}}\left(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma}\right)(\delta E^c_j\delta K^j_c) \\ &+ \frac{1}{\bar{p}^{3/2}}(\delta_{cd}\delta^{jk}\delta^{ef}\partial_eE^c_j\partial_fE^d_k)\bigg], \end{aligned} \tag{A7}$$

where for tensor modes

$$\delta E^a_i = -\tfrac{1}{2}\bar{p}h^a_i, \tag{A8}$$

$$\delta K^i_a = \frac{1}{2}\left[\dot{h}^i_a + \left(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma}\right)h^i_a\right]. \tag{A9}$$

Based on the Hamilton equations

$$\delta\dot{E}^a_i = \{\delta E^a_i, H^{\rm phen}_G + H_m\}, \tag{A10}$$

$$\delta\dot{K}^i_a = \{\delta K^i_a, H^{\rm phen}_G + H_m\}, \tag{A11}$$

we obtain the equation

$$\ddot{h}^{\ i}_{\ a} + 2\bar{k}\dot{h}^i_a - \nabla^2h^i_a + T_Qh^i_a = 16\pi G\Pi^{\ i}_{Qa}, \tag{A12}$$

where

$$T_Q = -2\left(\frac{\bar{p}}{\bar{\mu}}\frac{\partial\bar{\mu}}{\partial\bar{p}}\right)\bar{\mu}^2\gamma^2\left(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma}\right)^4, \tag{A13}$$

$$\Pi^{\ i}_{Qa} = \left[\frac{1}{3V_0}\frac{\partial\bar{H}_{\rm m}}{\partial\bar{p}}\left(\frac{\delta E^c_j\delta^j_a\delta^i_c}{\bar{p}}\right)\cos 2\bar{\mu}\gamma\bar{k} + \frac{\delta H_m}{\delta(\delta E^a_i)}\right] \tag{A14}$$

are quantum holonomy corrections.

We consider a homogeneous scalar field with the Hamiltonian

$$H_m = \int_\Sigma d^3\mathbf{x}\bar{N}\left(\frac{1}{2}\frac{\pi^2_\phi}{\sqrt{|\det E|}} + \sqrt{|\det E|}V(\phi)\right), \tag{A15}$$

where up to the second order

$$\begin{aligned} \sqrt{\det E} = \bar{p}^{3/2}\bigg[&1 + \frac{1}{2\bar{p}}\delta^i_a\delta E^a_i - \frac{1}{4\bar{p}^2}\delta^i_a\delta E^a_j\delta^j_b\delta E^b_i \\ &+ \frac{1}{8\bar{p}^2}\delta^i_a\delta E^a_i\delta^j_b\delta E^b_j\bigg], \end{aligned} \tag{A16}$$

$$\frac{1}{\sqrt{\det E}} = \frac{1}{\bar{p}^{3/2}}\bigg[1 - \frac{1}{2\bar{p}}\delta^i_a \delta E^a_i + \frac{1}{4\bar{p}^2}\delta^i_a \delta E^a_j \delta^j_b \delta E^b_i + \frac{1}{8\bar{p}^2}\delta^i_a \delta E^a_i \delta^j_b \delta E^b_j\bigg]. \tag{A17}$$

However, since $\delta^{ab}h_{ab} = 0 \Rightarrow \delta^i_a \delta E^a_i = 0$ the above expansion simplifies. Then

$$H_m = \bar{H}_m + \frac{1}{4}\int_\Sigma d^3\mathbf{x}\frac{\bar{N}}{\sqrt{\bar{p}}}\bigg(\frac{1}{2}\frac{\pi_\phi^2}{\bar{p}^3} - V(\phi)\bigg)\delta^i_a \delta E^a_j \delta^j_b \delta E^b_i + \mathcal{O}(E^3).$$

Now we can derive the variation

$$\frac{\delta H_m}{\delta(\delta E^a_i)} = \frac{1}{2}\frac{\bar{N}}{\sqrt{\bar{p}}}\bigg(\frac{1}{2}\frac{\pi_\phi^2}{\bar{p}^3} - V(\phi)\bigg)\delta^i_b \delta^k_a \delta E^b_k \tag{A18}$$

and the derivative

$$\frac{\partial \bar{H}_{\mathrm{m}}}{\partial \bar{p}} = \frac{3}{2}V_0\frac{\bar{N}}{\sqrt{\bar{p}}}\bigg(-\frac{1}{2}\frac{\pi_\phi^2}{\bar{p}^3} + V(\phi)\bigg). \tag{A19}$$

One can now easily find that in the classical limit, when we set $\cos(2\bar{\mu}\gamma\bar{k}) = 1$ in expression (A14), the source term vanishes. This is due to the opposite signs of the bracketed expression in Eqs. (A18) and (A19). When quantum holonomy corrections are present, we have

$$\cos(2\bar{\mu}\gamma\bar{k}) = 1 - 2\frac{\rho}{\rho_c}, \tag{A20}$$

which can be found from background equations of motion. Therefore the form of the quantum corrections simplifies to

$$T_Q = \frac{8\pi G}{3}\frac{\bar{p}\rho^2}{\rho_c}, \tag{A21}$$

$$\Pi^i_{Qa} = \Pi_Q h^i_a = \frac{1}{2}\bar{p}\frac{\rho}{\rho_c}(2V - \rho)h^i_a, \tag{A22}$$

where we have chosen $\bar{N} = \sqrt{\bar{p}}$ and adopted the expression (A8).

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