Emerging singularities in the bouncing loop cosmology

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In this paper we calculate $\mathcal{O}(\mu^4)$ corrections from holonomies in the loop quantum gravity, usually not taken into account. Allowance of the corrections of this kind is equivalent with the choice of the new quatization scheme. Quantization ambiguities in the loop quantum cosmology allow for this additional freedom and presented corrections are consistent with the standard approach. We apply these corrections to the flat Friedmann-Robertson-Walker cosmological model and calculate the modified Friedmann equation. We show that the bounce appears in the models with the standard $\mathcal{O}(\mu^2)$ quantization scheme and is shifted to the higher energies $\rho_{\text{bounce}} = 3\rho_{\text{c}}$. Also, a pole in the Hubble parameter appears for $\rho_{\text{pole}} = \frac{3}{2}\rho_{\text{c}}$ corresponding to *hyperinflation/deflation* phases. This pole represents a curvature singularity at which the scale factor is finite. In this scenario the singularity and bounce coexist. Moreover, we find that an ordinary bouncing solution appears only when quantum corrections in the lowest order are considered. Higher order corrections can lead to nonperturbative effects.

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I. INTRODUCTION

Strength of the gauge field F in some point x can be obtained from holonomy calculated around this point and taking the limit of the zero length of the loop. Loop quantum gravity (LQG) is kind of gauge theory describing gravitational degrees of freedom in terms of gauge field A which is elements of $\mathfrak{su}(2)$ algebra and conjugated variable E which is elements of $\mathfrak{su}(2)^*$ algebra [1]. To quantize this theory in a background independent way one introduces holonomies of the Ashtekar connection A

$$h_\alpha[A] = \mathcal{P}\exp\int_\alpha A \quad \text{where 1-form } A = \tau_i A^i_a dx^a, \quad (1)$$

where $\tau_i = -\frac{i}{2}\sigma_i$ (σ_i are Pauli matrices) and conjugated fluxes

$$F^i_S[E] = \int_S dF^i \quad \text{where 2-form } dF_i = \epsilon_{abc}E^a_i dx^b \wedge dx^c \quad (2)$$

as new fundamental variables [2,3]. Other variables like the field strength F should be expressed in term of these elementary variables. As we mentioned at the beginning the field strength can be expressed in terms of holonomies. However, another aspect of loop quantization starts to be important here. Namely, an area operator possesses a discrete spectrum with minimal nonzero eigenvalue Δ [4]. So we cannot simply shrink to zero the area enclosed by the loop. Instead of this we must stop the shrinking loop for a minimal value corresponding to the area gap Δ. This effect leads to quantum gravitational corrections to the expression for classical field strength. The expression for the field strength as a function of holonomies has a form [5]

$$F^k_{ab} = \lim_{\mu\to\bar{\mu}}\left\{-2\frac{\text{tr}[\tau_k(h^{(\mu)}_{\Box_{ij}} - \mathbb{I})]}{\mu^2 V_0^{2/3}}{}^o\omega^i_a{}^o\omega^j_b + \frac{\mathcal{O}(\mu^4)}{\mu^2}\right\}, \quad (3)$$

where the limit $\mu \to \bar{\mu}$ corresponds to the minimal value of the area gap Δ. However, this formula is adequate only when $\mathcal{O}(\mu^4)$ terms can be neglected, i.e., in the classical limit. In fact these terms, which form infinite series, are a function of F and A. The expression for the F as a function of holonomies should be therefore obtained by solving this equation in terms of the first factor on the right side. In the classical limit $\mu \to 0$ terms $\mathcal{O}(\mu^4)/\mu^2$ vanish and we recover a classical expression for the field strength. Until now in literature the first order quantum correction to field strength has been investigated. It means that terms $\mathcal{O}(\mu^4)$ have been neglected. This approach was dictated by the choice of the simplest quantization scheme. Namely, as it has been shown by Bojowald [6–8], the precise effective Hamiltonian must be a periodic function of the canonical variable c. The simplest form of this function we obtain when we perform the regularization of the expression for the classical field strength cutting off the terms $\mathcal{O}(\mu^4)$. This is a standard procedure in the loop quantum cosmology.

In this paper we calculate and study another nonvanishing contribution which is in fact a choice of the different

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regularization of the expression for the field strength. It means that we hold the $\mathcal{O}(\mu^4)$ factor, which is a function of F, and we solve equations for F as a function of the holonomies. This approach is equivalent to the choice of the new quantization scheme that is allowed due to quantization ambiguities.

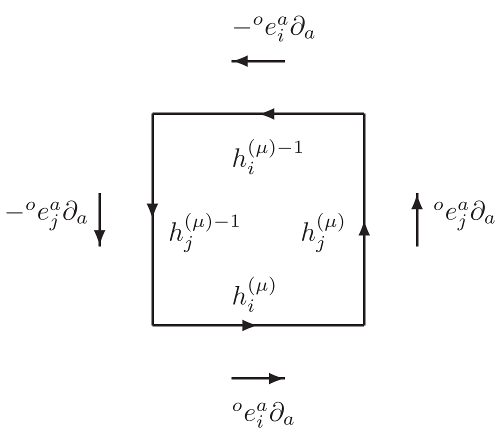
The organization of the text is the following. In Sec. II we calculate the expression for F as a function of holonomies in $\mathcal{O}(\mu^4)$ order. Then in Sec. III we apply this result to the flat Friedmann-Robertson-Walker (FRW) cosmological model. We show that obtained corrections have important influence for this model. In Sec. V we summarize the results. Finally in the appendix we give some basics of loop quantum cosmology connected with the subject of this paper and explain the employed notation.

II. HOLONOMY CORRECTIONS

From the definition (1) we can calculate holonomy for a homogeneous model in the particular direction ${}^oe^a_i\partial_a$ and the length $\mu V_0^{1/3}$

$$h_i^{(\mu)} = e^{\tau_i \mu c} = \mathbb{I}\cos\left(\frac{\mu c}{2}\right) + 2\tau_i \sin\left(\frac{\mu c}{2}\right). \tag{4}$$

From such particular holonomies we can construct a holonomy along the closed curve $\alpha = \Box_{ij}$ as schematically presented in the diagram below



which can be written as

$$h^{(\mu)}_{\Box_{ij}} = h_i^{(\mu)} h_j^{(\mu)} h_i^{(\mu)-1} h_j^{(\mu)-1} = e^{\mu B_i} e^{\mu B_j} e^{-\mu B_i} e^{-\mu B_j} \tag{5}$$

where we have introduced

$$B_i := V_0^{1/3} A_a{}^o e^a_i = V_0^{1/3} c \tau_i. \tag{6}$$

Factors B_i are elements of $\mathfrak{su}(2)$ algebra so to perform a product of exponents in Eq. (5) we need to use the Baker-Campbell-Hausdorff formula

$$e^X e^Y = \exp\{X + Y + \tfrac{1}{2}[X,Y] + \tfrac{1}{12}([X,[X,Y]] + [Y,[Y,X]]) - \tfrac{1}{24}[Y,[X,[X,Y]]] + \ldots\}. \tag{7}$$

To calculate the $\mathcal{O}(\mu^4)$ correction the elements of the expansion written above are sufficient. Applying this formula to Eq. (5) we obtain

$$\begin{aligned} h^{(\mu)}_{\Box_{ij}} &= \exp\Big\{\mu^2[B_i,B_j] + \frac{\mu^3}{2}[B_i + B_j,[B_i,B_j]] \\ &\quad - \frac{\mu^4}{12}[B_j,[B_i,[B_i,B_j]]] + \frac{\mu^4}{6}[B_i + B_j,[B_i \\ &\quad + B_j,[B_i,B_j]]] + \mathcal{O}(\mu^5)\Big\} \\ &= \mathbb{I} + \mu^2[B_i,B_j] + \frac{\mu^3}{2}[B_i + B_j,[B_i,B_j]] \\ &\quad - \frac{\mu^4}{12}[B_j,[B_i,[B_i,B_j]]] + \frac{\mu^4}{6}[B_i + B_j,[B_i \\ &\quad + B_j,[B_i,B_j]]] + \frac{\mu^4}{2}[B_i,B_j][B_i,B_j] + \mathcal{O}(\mu^5). \end{aligned} \tag{8}$$

Now, multiplying this expression by τ_k, using definition (6), and taking a trace of both sides we obtain

$$\begin{aligned} \mathrm{tr}[\tau_k(h^{(\mu)}_{\Box_{ij}} - \mathbb{I})] &= \mu^2 c_{\mathrm{h}}^2 \epsilon_{ijl}\,\mathrm{tr}(\tau_k\tau_l) + \frac{\mu^3}{2} c_{\mathrm{h}}^3 \epsilon_{ijl}(\epsilon_{ilm} + \epsilon_{jlm}) \\ &\quad \times \mathrm{tr}(\tau_k\tau_m) - \frac{\mu^4}{12} c_{\mathrm{h}}^4 \epsilon_{ijl}\epsilon_{ilm}\epsilon_{jmn}\,\mathrm{tr}(\tau_k\tau_n) \\ &\quad + \frac{\mu^4}{6} c_{\mathrm{h}}^4 \epsilon_{ijl}(\epsilon_{ilm} + \epsilon_{jlm})(\epsilon_{imn} + \epsilon_{jmn}) \\ &\quad \times \mathrm{tr}(\tau_k\tau_n) + \frac{\mu^4}{2} c_{\mathrm{h}}^4 \epsilon_{ijl}\epsilon_{ijm}\,\mathrm{tr}(\tau_k\tau_l\tau_m). \end{aligned} \tag{9}$$

We mention that $\{ijk\}$ are external indices and the Einstein summation convention is not fulfilled. The introduced parameter c_{h} corresponds to the effective canonical variable c which is expressed as a function of holonomies. With use of Eq. (4) we can directly calculate the left side of Eq. (9); we obtain

$$\mathrm{tr}[\tau_k(h^{(\mu)}_{\Box_{ij}} - \mathbb{I})] = -\frac{\epsilon_{kij}}{2}\sin^2(\mu c). \tag{10}$$

Then, using properties of τ_i matrices we obtain

$$\tfrac{1}{3}\mu^4 c_{\mathrm{h}}^4 - \mu^2 c_{\mathrm{h}}^2 + \sin^2(\mu c) = 0. \tag{11}$$

The $\mathcal{O}(\mu^3)$ order contribution simply vanishes. The solutions of this equation have a form

$$c^2_{\mathrm{h}\pm} = \frac{1 \pm \sqrt{1 - \frac{4}{3}\sin^2(\mu c)}}{\frac{2}{3}\mu^2}. \tag{12}$$

When we expand the square in the solution for $c^2_{\mathrm{h}-}$ we obtain

$$c^2_{\mathrm{h}-} = \left[\frac{\sin(\mu c)}{\mu}\right]^2 + \frac{1}{3}\frac{\sin^4(\mu c)}{\mu^2} + \ldots. \tag{13}$$

The first factor of the expansion corresponds to the known case when $\mathcal{O}(\mu^4)$ corrections are ignored. We can easily check then the classical limit $\mu \to 0$, $c_{\rm h}^2 \to c^2$ is recovered only in the $c_{\rm h-}^2$ case. The case $c_{\rm h+}^2$ should be therefore treated as unphysical. However, as we will see in the next section, both solutions lead to the same modified Friedmann equation. So we can keep both solutions.

Finally, the expression for the effective field strength has a form

$$F^k_{ab} = \epsilon^k_{ij} \frac{1 \pm \sqrt{1 - \frac{4}{3}\sin^2(\bar{\mu}c)}}{\frac{2}{3}\bar{\mu}^2 V_0^{2/3}} {}^o\omega^i_a {}^o\omega^j_b. \quad (14)$$

We have performed here the limit $\mu \to \bar{\mu}$ where

$$\bar{\mu} = \sqrt{\frac{\Delta}{|p|}}. \quad (15)$$

For details of this limit see Ref. [5] or appendices to Refs. [9–11].

As we mentioned earlier the precise effective Hamiltonian must be a periodic function of c. In our case the effective Hamiltonian has a form $H_{\rm eff} \sim \sqrt{|p|}c_{\rm h-}^2$ where the $c_{\rm h-}^2$ can be expressed as

$$c_{\rm h-}^2 = \frac{1}{2\bar{\mu}^2} \sum_{n=1}^{\infty} \frac{(2n)!}{(2n-1)n!^2 3^{n-1}(2i)^{2n}} \times [\exp(i\bar{\mu}c) - \exp(-i\bar{\mu}c)]^{2n}. \quad (16)$$

As we see this function is periodic and forms an infinite series numerated by integers. However, this infinity is allowed in the frames of loop quantum cosmology. The obtained effective Hamiltonian is correct; however, it is not given by the simple function as we should expect for fundamental expressions. Thus, we should keep in mind that we are looking for the effective Hamiltonian and there are no circumstances that such a Hamiltonian must have a mathematically simple allowed form.

In the next section we will use the calculated effective field strength F for the FRW $k = 0$ cosmological model.

III. APPLICATION TO FRW $k = 0$

With use of Eq. (14) we can derive the effective Hamiltonian for the flat FRW model in the form

$$H_{\rm eff} = -\frac{3}{8\pi G\gamma^2} \frac{1 \pm \sqrt{1 - \frac{4}{3}\sin^2(\bar{\mu}c)}}{\frac{2}{3}\bar{\mu}^2} \sqrt{|p|} + |p|^{3/2}\rho. \quad (17)$$

For details we refer the reader to the appendix. This Hamiltonian fulfils the so-called Hamiltonian constraint $H_{\rm eff} = 0$. From Hamilton equations we can calculate the evolution of the canonical variable p

$$\dot{p} = \{p, H_{\rm eff}\} = -\frac{8\pi G\gamma}{3} \frac{\partial H_{\rm eff}}{\partial c} \quad (18)$$

and with use of (17) we obtain

$$\dot{p} = \mp \frac{\sqrt{|p|}}{\gamma\bar{\mu}} \frac{2\sin(\bar{\mu}c)\cos(\bar{\mu}c)}{\sqrt{1 - \frac{4}{3}\sin^2(\bar{\mu}c)}}. \quad (19)$$

Applying Eq. (19), the Hamiltonian constraint $H_{\rm eff} = 0$, and definition of the Hubble parameter $H = \frac{\dot{p}}{2p}$, we finally derive the modified Friedmann equation

$$H^2_{\mathcal{O}(\mu^4)} = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{3\rho_{\rm c}}\right)\left[\frac{3}{4} + \frac{1}{4}\frac{1}{(1 - \frac{2}{3}\frac{\rho}{\rho_{\rm c}})^2}\right], \quad (20)$$

where we have introduced

$$\rho_{\rm c} = \frac{\sqrt{3}}{16\pi^2\gamma^3 l_{\rm Pl}^4}. \quad (21)$$

As we see the obtained equation does not depend on the sign $\pm$ in the Hamiltonian. An analogous equation in the lowest order has been calculated earlier [5] and has a form

$$H^2_{\mathcal{O}(\mu^2)} = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_{\rm c}}\right). \quad (22)$$

This equation leads to the bounce for $\rho = \rho_{\rm c}$. An analogous bounce is also present in the derived model (20); however, now the bounce is shifted to the higher energy densities

$$\rho_{\rm bounce} = 3\rho_{\rm c}. \quad (23)$$

Another important property is the pole in the Hubble parameter for

$$\rho_{\rm pole} = \tfrac{3}{2}\rho_{\rm c} \quad (24)$$

as we see from Eq. (20). We show these features in Fig. 1. In the upper panel we present H^2 as a function of energy density for $\mathcal{O}(\mu^2)$ and $\mathcal{O}(\mu^4)$ cases. In the lower panel we compare the evolution of the Hubble parameter as a function of p for the radiation dominated universe ($\rho \propto 1/p^2$).

In both cases we observe a nonperturbative feature, namely, the pole for $\rho = \frac{3}{2}\rho_{\rm c}$. This fact indicates that higher order corrections from holonomies can have important influence for dynamical behavior for small values of p. It is clear from a parameter of expansion (15) which grows for small values of p. For large p the classical case is clearly recovered; however, behavior for small values of p is highly complicated. Namely, as our study suggests, higher order terms of expansion have nonperturbative influence for dynamics and this fact can seriously complicate a simple bouncing universe picture. We investigate this issue in the next section.

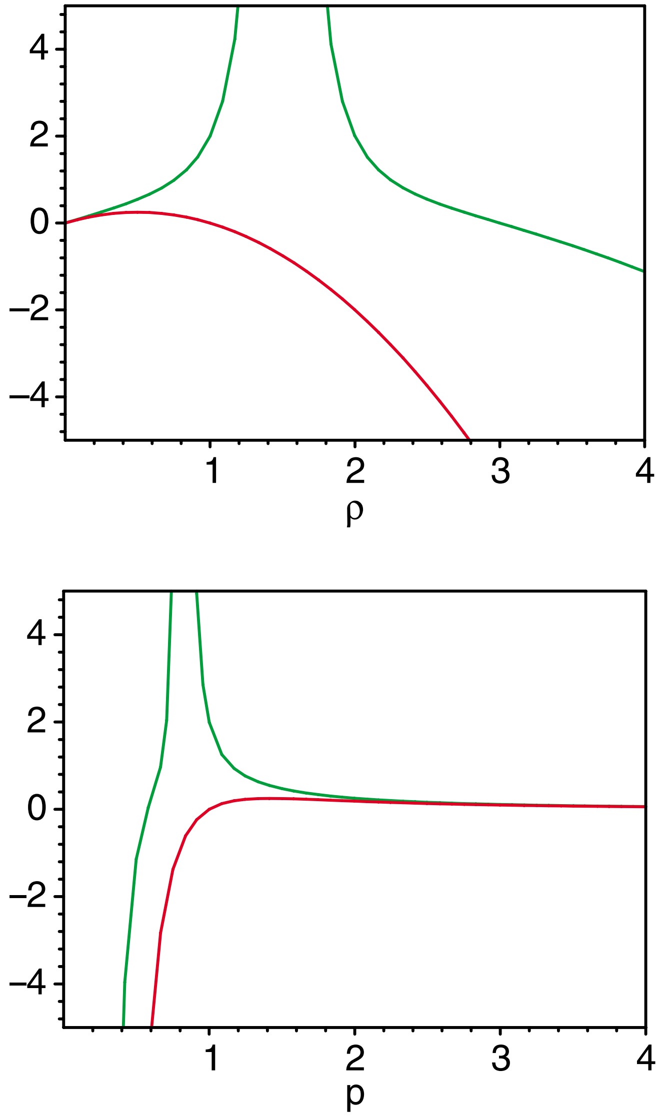


FIG. 1 (color online). *Upper*: Evolution of $H^2_{\mathcal{O}(\mu^2)}$ (red, bottom curve) and $H^2_{\mathcal{O}(\mu^4)}$ (green, top curve) as a function of energy density ρ. The parts below zero value of H^2 are unphysical. The ρ is in units of ρ_c. *Lower*: Evolution of $H^2_{\mathcal{O}(\mu^2)}$ (red, bottom curve) and $H^2_{\mathcal{O}(\mu^4)}$ (green, top curve) as a function of p for the model with radiation $(\rho \propto 1/p^2)$. On both panels a physically admissible region corresponds to $H^2 \geq 0$.

IV. QUALITATIVE ANALYSIS OF DYNAMICS

The advantage of qualitative methods of analysis of differential equations [12] is that we obtain all evolutional paths for all admissible initial conditions. In this approach the evolution of the system is represented by trajectories in the phase space and asymptotic states by critical points. We demonstrate that dynamics of the model can be reduced to a two-dimensional autonomous dynamical system. These methods allow one to distinguish a generic evolutional scenario.

In Fig. 2 we show the phase portrait for all admissible initial conditions (all values of total energy E of the fictitious particle moving in a one-dimensional potential proportional to p^3) for the model with the free scalar field.

The physical trajectories are situated in the region at which $E - V$ is nonnegative or p is larger than some minimal value and p' is zero. The physical trajectories lie in the nonshaded region bounded by a zero velocity curve which represents a homoclinic orbit. Of course the whole system is symmetric with respect to the reflection (H is changed in $-H$). The vertical (blue) line on the phase portrait represents points at which trajectories pass horizontally through the inflection point (*hyperinflationary/deflationary* phases). Therefore, evolution comprises the bounce solution interpolating static phases of evolution, see Fig. 3. There is the intermediate phase of evolution at which we have the inflection point at the diagram of $p(t)$. Note that it is the singularity state with rapid growth of the scale factor, we call this phase the hyperinflation. It is important to note that energy density is finite during transition through these singularities. Similar finite scale factor singularities have been studied recently by Cannata *et al.* [13].

In the phase diagram in Fig. 2 we adjoint a circle at infinity in a standard way via Poincaré construction. Note that all trajectories are starting from an unstable node—representing a static Einstein universe and landing at a stable node representing another static Einstein universe.

Parisi *et al.* [14] pointed out that stability of Einstein static models in high-energy modifications of general relativity is important from the point of view of the so-called emergent universe scenario [15]. Mulryne *et al.* [16] investigated the stability of the Einstein static model and they found that the LQC Einstein static model is representing a center type of critical point on the phase portrait. As it is well known, such a critical point is structurally unstable. Note that in our case this static universe is representing a

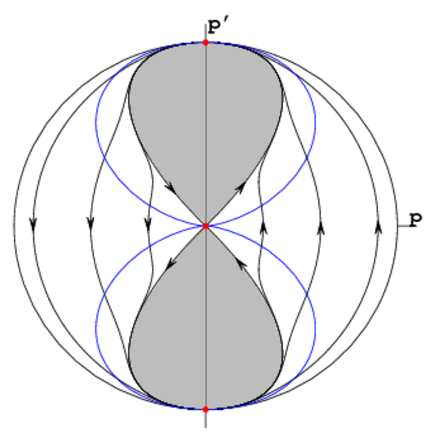


FIG. 2 (color online). The phase portrait for all admissible initial conditions. The vertical (blue) line is representing points at which trajectories pass horizontally through the inflection point (*hyperinflationary/deflationary* phases).

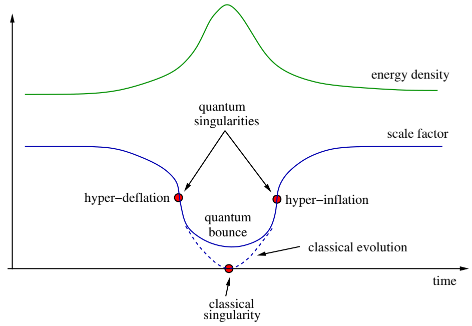


FIG. 3 (color online). The schematic picture of the evolution of the model. The top curve (green) represents the energy density of the free scalar field. It is worth noting that this energy density is finite during whole evolution, even during transitions through singularities. The bottom curve (blue) presents schematic evolution of the scale factor for the investigated model. The dashed curve represents the classical evolution which is not realized in the presented quantum model.

node type of critical point. This modification of stability in the presented model has important consequences for the emergent universe scenario, since as it is well known in general relativity, a static universe is unstable and is represented by a saddle type of critical point and therefore it requires fine-tuning. Moreover, corrections considered lead to a regularization of the big-bang singularity [17].

V. SUMMARY

In this paper we have calculated another nonvanishing contribution to the quantum holonomy correction in loop quantum gravity. Quantum correction of this kind appears when we express the Ashtekar connection A and field strength F in terms of holonomies. The source of quantum modification to classical expressions is a nonvanishing area enclosed by a loop as the result of existence of the area gap Δ.

We had applied obtained corrections to the flat FRW cosmological model and then we calculated the resulting quantum gravitational modifications to the Friedmann equation. The holonomy correction in the lowest order to the flat FRW model was calculated earlier [5,18,19] and extensively studied [10,20]. These investigations uncovered existence of the bounce for energy scales ρ_c. In this picture the standard big-bang singularity is replaced by the nonsingular big bounce. Calculations performed in the present paper indicate that the holonomy correction in the next nonvanishing order holds this picture. Namely, the initial singularity is still preserved. However the bounce appears now for higher energy density $\rho_{\mathrm{bounce}} = 3\rho_c$. Another important new feature is the appearance of a pole in the Hubble parameter for $\rho_{\mathrm{pole}} = \frac{3}{2}\rho_c$ corresponding to *hyperinflationary/deflationary* phases. This leads to more complicated dynamical behavior at these energy scales.

We showed that the generic evolutional scenario for the model with the free scalar field starts from the static Einstein universe then recollapses passing through the curvature singularity with a finite scale factor (hyperdeflation) towards the bounce and goes in the expanding phase through the second curvature singularity (hyperinflation) and ends in the static Einstein universe. During the transition through the singularities the universal critical behavior $H \propto |\rho - \rho_{\mathrm{pole}}|^{-1}$ holds. Therefore, in the presented scenario the bounce connects these two finite scale factor singularities.

As we see, the higher order quantum correction in LQG can have an important influence on dynamical behavior of cosmological models. It is not unlikely that the nonsingular bounce which appeared in the lowest order can be only an artifact of simplifications and can disappear when the whole contribution will be taken into account. Further investigations of quantum corrections from LQG are still necessary. We conclude that higher order holonomy corrections and resulting different quantization schemes should be also seriously taken into account in considerations.

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APPENDIX: FLAT FRW MODEL IN LOOP QUANTUM GRAVITY

The FRW $k = 0$ spacetime metric can be written as

$$ds^2 = -N^2(x)dt^2 + q_{ab}dx^a dx^b \tag{A1}$$

where $N(x)$ is the lapse function and the spatial part of the metric is expressed as

$$q_{ab} = \delta_{ij}\omega^i_a\omega^j_b = a^2(t)^o q_{ab} = a^2(t)\delta_{ij}{}^o\omega^i_a{}^o\omega^j_b. \tag{A2}$$

In this expression ${}^o q_{ab}$ is a fiducial metric and ${}^o\omega^i_a$ are co-triads dual to the triads ${}^o e^a_i$, ${}^o\omega^i({}^o e_j) = \delta^i_j$ where ${}^o\omega^i = {}^o\omega^i_a dx^a$ and ${}^o e_i = {}^o e^a_i \partial_a$. From these triads we construct the Ashtekar variables

$$A^i_a \equiv \Gamma^i_a + \gamma K^i_a = cV_0^{-1/3}{}^o\omega^i_a, \tag{A3}$$

$$E^a_i \equiv \sqrt{|\det q|}e^a_i = pV_0^{-2/3}\sqrt{{}^o q}\,{}^o e^a_i, \tag{A4}$$

where

$$|p| = a^2 V_0^{2/3}, \tag{A5}$$

$$c = \gamma \dot{a} V_0^{1/3}. \tag{A6}$$

Note that the Gaussian constraint implies that $p \leftrightarrow -p$ leads to the same physical results. The factor γ is called the Barbero-Immirzi parameter, $\gamma = \ln 2/(\pi\sqrt{3})$. In the definition (A3) the spin connection is defined as

$$\Gamma_a^i = -\epsilon^{ijk} e_j^b (\partial_{[a} e_{b]}^k + \tfrac{1}{2} e_k^c e_a^l \partial_{[c} e_{b]}^l), \tag{A7}$$

and the extrinsic curvature is defined as

$$K_{ab} = \frac{1}{2N}[\dot{q}_{ab} - 2D_{(a}N_{b)}] \tag{A8}$$

which corresponds to $K_a^i := K_{ab} e_i^b$.

The scalar constraint, in the Ashtekar variables, has the form

$$H_G = \frac{1}{16\pi G}\int_\Sigma d^3x N(x) \frac{E_i^a E_j^b}{\sqrt{|\det E|}} \times [\varepsilon_{k\,k}^{ij} F_{ab}^k - 2(1+\gamma^2) K_{[a}^i K_{b]}^j], \tag{A9}$$

where field strength is expressed as

$$F_{ab}^k = \partial_a A_b^k - \partial_b A_a^k + \epsilon^k{}_{ij} A_a^i A_b^j. \tag{A10}$$

With use of (A3), (A4), and (A10) the Hamiltonian (A9) assumes the form

$$H_G = -\frac{3}{8\pi G \gamma^2}\sqrt{|p|} c^2, \tag{A11}$$

where we have assumed a gauge of $N(x) = 1$. Quantum corrections to this Hamiltonian come when we express $\sqrt{|p|}$ and c^2 in terms of background-independent variables. In this paper we have concentrated on the corrections to the factor c^2, called holonomy corrections. For a short review of quantum corrections we refer the reader to the appendix of Ref. [11].

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