W stronę Ery Plancka

Efekty obserwacyje Pętlowej Teorii Grawitacji i ich testowanie

Jakub Mielczarek

12 Czerwca, 2007

Horyzont

$$ds^2 = -a^2(\tau)\left[d\tau^2 - \frac{dr^2}{1-kr^2} - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$$

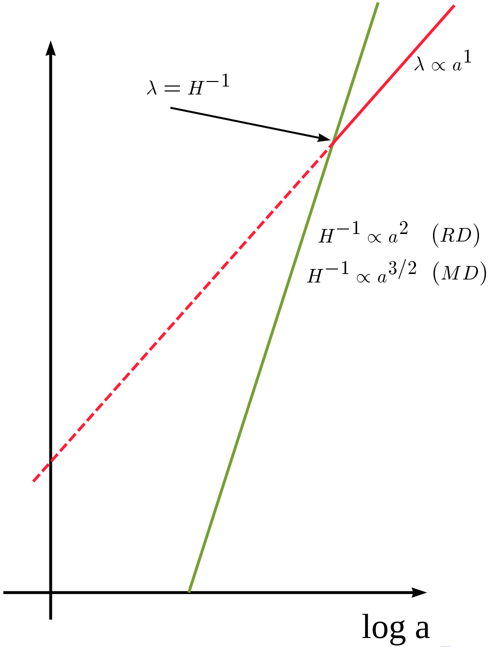
$$d\tau = \frac{dt}{a} \quad \text{czas konforemny}$$

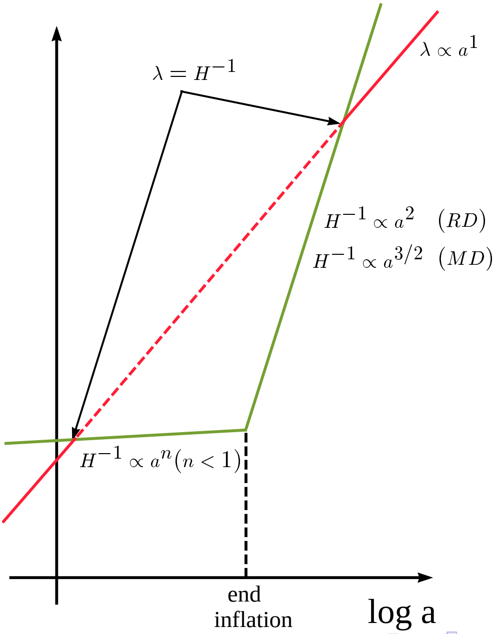
Odległość fizyczna $R = a \cdot r$

$$ds^2 = 0 \Rightarrow dr = \frac{dt}{a(t)} \Rightarrow r_H(t) = \int_0^t \frac{dt'}{a(t')} \Rightarrow R_H(t) = a(t)\int_0^t \frac{dt'}{a(t')}$$

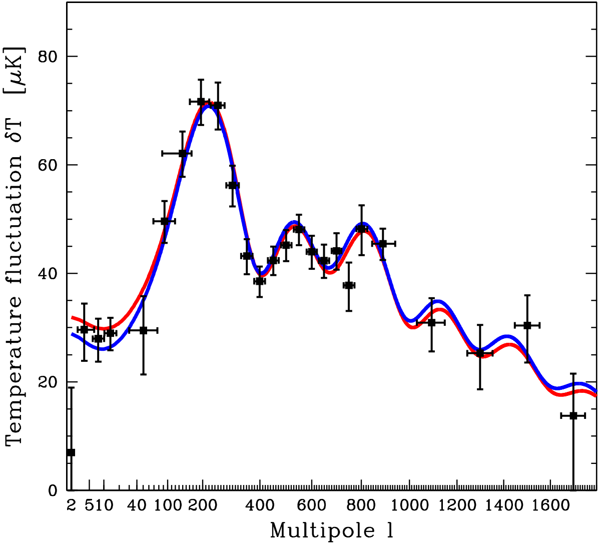
dla $a(t) \propto t^n$

$$R_H(t) = a(t)\int_0^t \frac{dt'}{a(t')} = \frac{t}{1-n} = n\frac{H^{-1}}{1-n} \sim H^{-1}$$


$\lambda = H^{-1}$
$\lambda \propto a^1$
$H^{-1} \propto a^2 \quad (RD)$
$H^{-1} \propto a^{3/2} \quad (MD)$
log a



$\lambda = H^{-1}$
$\lambda \propto a^1$
$H^{-1} \propto a^2 \quad (RD)$
$H^{-1} \propto a^{3/2} \quad (MD)$
$H^{-1} \propto a^n (n < 1)$
end inflation
log a


Temperature fluctuation δT [μK]
80
60
40
20
0
2 510 40 100 200 400 600 800 1000 1200 1400 1600
Multipole l

Pole Inflatonu

$$\mathcal{S} = \int d^4x\sqrt{-g}\mathcal{L} = \int d^4x\sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi)\right]$$

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2\phi}{a^2} + \frac{dV}{d\phi} = 0$$

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\mathcal{L}$$

$$\begin{aligned} T_{00} = \rho_\phi &= \frac{\dot{\phi}^2}{2} + V(\phi) + \frac{(\nabla\phi)^2}{2a^2} \\ T_{ii} = p_\phi &= \frac{\dot{\phi}^2}{2} - V(\phi) - \frac{(\nabla\phi)^2}{6a^2} \end{aligned}$$

Slow-Roll Infation

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

$$H^2 = \frac{8\pi}{3m_p^2}\left[\frac{\dot{\phi}^2}{2} + V(\phi)\right]$$

$\dot{\phi}^2 \ll V$

$$H^2 \simeq \frac{8\pi}{3m_p^2} V(\phi)$$

$$3H\dot{\phi} \simeq -\frac{dV}{d\phi}$$

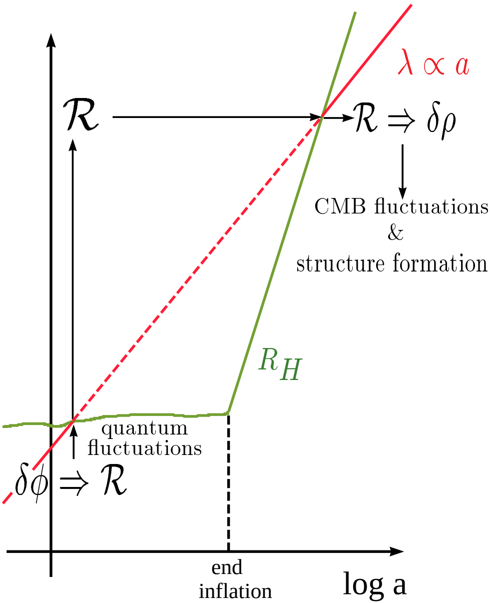
Reheating

$$\begin{aligned} \rho_I &\quad - \quad \text{całkowita gęstość energii inflacji} \\ a_I &\quad - \quad \text{początek koherentnych oscylacji} \end{aligned}$$

Pamiętamy że $t = nH^{-1}$.

$$\begin{aligned} \rho_\phi &= \rho_I \left(\frac{a_I}{a}\right)^3 \to \rho_R = \frac{\pi^2}{30} g_* T_{RH}^4 \\ H^2 &= \frac{8\pi}{3m_p^2} \rho_I \left(\frac{a_I}{a}\right)^3 \end{aligned}$$

$$T_{RH} = \left(\frac{90}{8\pi^3 g_*}\right)^{1/4} \sqrt{\Gamma_\phi m_p}$$


$\lambda \propto a$
$\mathcal{R}$
$\mathcal{R} \Rightarrow \delta\rho$
CMB fluctuations
&
structure formation
R_H
quantum
fluctuations
$\delta\phi \Rightarrow \mathcal{R}$
end
inflation
log a

Semi-klasyczna dynamika

$$\mathcal{S} = \int d^4x\sqrt{-g}\mathcal{L} = \int d^4x\sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi)\right]$$

Dla płaskiego wszechswiata FRW $\sqrt{-g} = a^3$

$$T_{00} = \mathcal{H} = \frac{\dot{\phi}^2}{2} + V(\phi) + \frac{(\nabla\phi)^2}{2a^2}$$

Pęd kanoniczny $\pi_\phi = \dot{\phi}a^3$.

$$H_\phi = \int dx^3 a^3\mathcal{H} = \int dx^3 a^3\left[\frac{\dot{\phi}^2}{2} + V(\phi) + \frac{(\nabla\phi)^2}{2a^2}\right]$$

$$H_\phi = a^{-3}\int dx^3\left[\frac{\pi_\phi^2}{2}\right] + a\int dx^3\left[\frac{(\nabla\phi)^2}{2}\right] + a^3\int dx^3\left[V(\phi)\right]$$

$$H_\phi = d(a)\int dx^3 \left[\frac{\pi_\phi^2}{2}\right] + a\int dx^3 \left[\frac{(\nabla\phi)^2}{2}\right] + a^3 \int dx^3 \left[V(\phi)\right]$$

gdzie

$$d(a) = \frac{D(q)}{a^3}, \;\; q = \frac{a^2}{a_*^2}, \;\; a_* = \sqrt{\frac{\gamma j}{3}} l_{Pl}$$

oraz

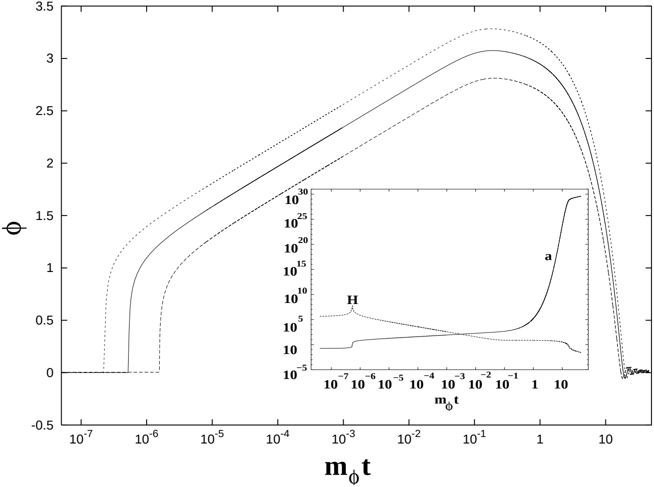
$$D(q) = q^{-3/2}\left\{\frac{3}{2l}\left(\frac{1}{l+2}\left[(q+1)^{l+2} - |q-1|^{l+2}\right] - \frac{q}{1+l}\left[(q+1)^{l+1} - \mathrm{sgn}(q-1)|q-1|^{l+1}\right]\right)\right\}^{3/(2-2l)}.$$

Rówanie ruchu:

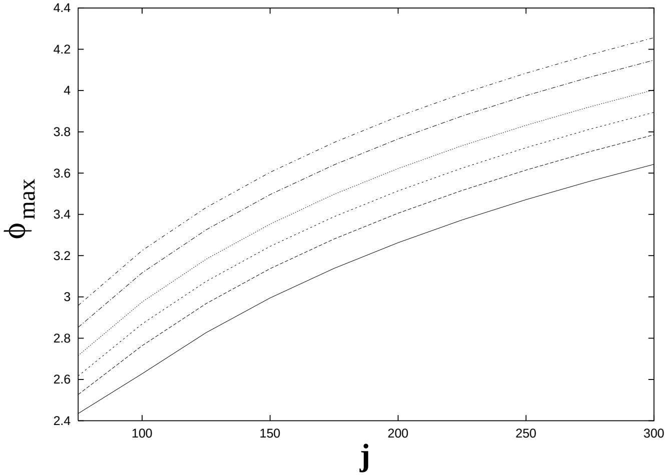
$$\ddot{\phi} + \left(3H\dot{\phi} - \frac{\dot{D}(a)}{D(a)}\right) + D(a)\frac{dV}{d\phi} = 0$$

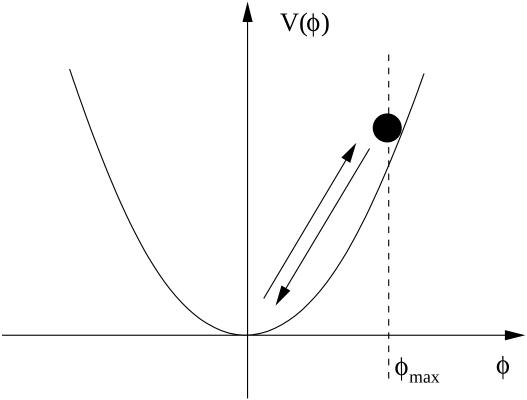
Równania Friedmana i Raychaudhuriego:

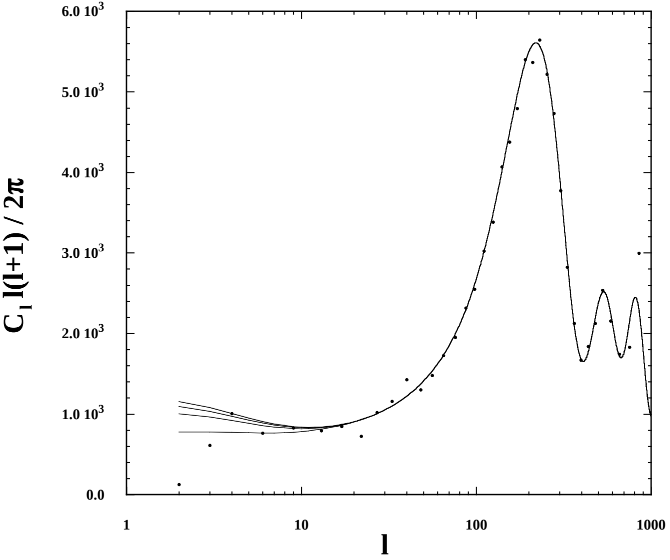
$$\begin{aligned} H^2 &= \frac{8\pi G}{3}\left[\frac{\dot{\phi}^2}{2D(a)} + V(\phi)\right] \\ \frac{\ddot{a}}{a} &= -\frac{8\pi G}{3}\left[\frac{\dot{\phi}^2}{D(a)}\left(1 - \frac{\dot{D}(a)}{4HD(a)}\right) - V(\phi)\right] \end{aligned}$$



φ
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 10
$m_\phi t$
a
H
10^{30} 10^{25} 10^{20} 10^{15} 10^{10} 10^{5} 10 10^{-5}
10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 10
$m_\phi t$


4.4
4.2
4
3.8
3.6
3.4
3.2
3
2.8
2.6
2.4
100
150
200
250
300
ϕ_{max}
j


V(ϕ)
ϕ_max
ϕ


6.0 10^3
5.0 10^3
4.0 10^3
3.0 10^3
2.0 10^3
1.0 10^3
0.0
C_l l(l+1) / 2π
1
10
100
1000
l

Index widmowy ma postać [arXiv:astro-ph/0311015]

$$n_s = 1 + \frac{1}{1+\epsilon_1}\left(6\epsilon_1 - \frac{2\dot{\phi}}{H\phi}\right)$$

gdzie

$$\epsilon_1 \equiv \frac{\dot{H}}{H^2}.$$

Jesteśmy w rejonie dominacji energii potencjalej $\dot{\phi}^2/2 \ll V$ więc

$$w = \frac{\dot{\phi}^2/2 - V}{\dot{\phi}^2/2 + V} \simeq \text{const} \simeq -1.$$

Stąd biorąc

$$V = \frac{1}{2}m^2\phi^2$$

mamy

$$\frac{\dot{\phi}}{\phi} = \sqrt{\frac{1+w}{1-w}}m.$$

Wyliczmy teraz

$$\epsilon_1 \equiv \frac{\dot{H}}{H^2} = \frac{\frac{\ddot{a}}{a} - H^2}{H^2} = \frac{\ddot{a}}{aH^2} - 1$$

Korzystając z rówan Friedmana dostajemy

$$\epsilon_1 = \frac{\ddot{a}}{aH^2} - 1 = \frac{-\frac{8\pi G}{6}(\rho + 3p)}{\frac{8\pi G}{3}\rho} - 1 = -\frac{1}{2}(1 + 3w) - 1.$$

Stąd dostajemy

$$n_s = 1 + \frac{18(1+w)}{1+3w} + \sqrt{\frac{1+w}{1-w}}\frac{1}{1+3w}\frac{m}{H}$$

[arXiv:gr-qc/0411012]

$$n_s = 1 + C_w(-n-2)\left(\frac{k}{2\pi H}\right)^{2(1-n)} + 4c_0(1-n)\left(\frac{k}{2\pi H}\right)^{4(1-n)}$$

gdzie $-\infty < n < -2$.

$$\begin{aligned} n &= -\frac{1}{2}\left(1+\frac{3}{1-l}\right) \\ C_w &= 2\frac{1+w}{1-w} \\ c_0 &= \frac{\pi^2}{\sigma^2}\left[1+\frac{\sigma^2}{3(1+\sigma)}\right] \\ \sigma &= 4\pi\left(\frac{2\pi}{3}\right)^{2/3}\left(\frac{\bar{\rho}}{m_p^4}\right)^{1/3} \end{aligned}$$

$$
\begin{aligned}
n_s &= 1 + C_w(-n-2)\left(\frac{k}{2\pi H}\right)^{2(1-n)} + c_0 4(1-n)\left(\frac{k}{2\pi H}\right)^{4(1-n)} \\
\alpha_s &= C_w(-n-2)2(1-n)\left(\frac{k}{2\pi H}\right)^{2(1-n)} + c_0 4^2(1-n)^2\left(\frac{k}{2\pi H}\right)^{4(1-n)} \\
\beta_s &= C_w(-n-2)4(1-n)^2\left(\frac{k}{2\pi H}\right)^{2(1-n)} + c_0 4^3(1-n)^3\left(\frac{k}{2\pi H}\right)^{4(1-n)}
\end{aligned}
$$

Dane

SDSS [arXiv:astro-ph/0608632]

$$\begin{aligned} n_s &= 0.953^{+0.016}_{-0.016} \quad \text{dla} \quad k^* = 0.05/Mpc \\ \alpha_s = \frac{dn_s}{d\ln k} &= -0.040^{+0.027}_{-0.027} \end{aligned}$$

WMAP+SDSS [arXiv:astro-ph/0703625]

$$\begin{aligned} n_s &= 1.03 \pm 0.05 \quad \text{dla} \quad k^* = 0.01/Mpc \\ \alpha_s = \frac{dn_s}{d\ln k} &= -0.07 \pm 0.04 \\ \beta_s = \frac{d\alpha_s}{d\ln k} &= -0.04 \pm 0.04 \end{aligned}$$

Operator odwrotnej objętości

$$\widehat{d(a)}^{(j,l)}_{\mu} = \left(\frac{9}{l_{pl}^2 lj(j+1)(2j+j)} \sum_{k=-j}^{j} k|p_{\mu+2k}|^l \right)^{3/(2-2l)}$$

Efekty kwantowe zaczynają być istotne przy

$$a_* = \sqrt{\frac{\gamma j}{3}} l_{pl}$$

gdzie γ to parametr Barbero-Immirzi

$$\gamma = \frac{\ln 2}{\pi\sqrt{3}}$$

wyznaczony ze zgodności dla entropii czarnych dziur.

$$d_j(a) = D(q)\frac{1}{a^3}$$

gdzie q jest zdefiniowane jako

$$q \equiv \frac{a^2}{a_*^2}$$

dla semi-klasycznego wszechświata ($l_{pl} < a \ll a_*$) poprawka kwantowa ma postać

$$\begin{aligned} D(q) &= q^{-3/2}\left\{\frac{3}{2l}\left(\frac{1}{l+2}\left[(q+1)^{l+2} - |q-1|^{l+2}\right]\right.\right. \\ &- \left.\left.\frac{q}{1+l}\left[(q+1)^{l+1} - \mathrm{sgn}(q-1)|q-1|^{l+1}\right]\right)\right\}^{3/(2-2l)} . \end{aligned}$$

$$\mathcal{H} = \frac{1}{2} d_j(a) p_\phi^2 + a^3 V(\phi) \quad \text{gdzie} \quad p_\phi = d_j^{-1}(a)\dot{\phi}$$

To prowadzi do równania ruchu

$$\ddot{\phi} + \left(3H\dot{\phi} - \frac{\dot{D}}{D}\right) + D\frac{dV}{d\phi} = 0$$

Równaie Friedmana i Raychaudhuriego dla wszechświata z polem skalarnym mają postać

$$\begin{aligned} H^2 &= \frac{8\pi G}{3}\left[\frac{\dot{\phi}^2}{2D} + V(\phi)\right] \\ \frac{\ddot{a}}{a} &= -\frac{8\pi G}{3}\left[\frac{\dot{\phi}^2}{D}\left(1 - \frac{\dot{D}}{4HD}\right) - V(\phi)\right]. \end{aligned}$$

$$\dot{H} = -4\pi G \frac{\dot{\phi}^2}{D}\left(1 - \frac{\dot{D}}{6HD}\right).$$

Dzięki poprawce kwantowej D w obszarze semiklasycznym ($l_{pl} < a \ll a_*$), wyrażenie w nawiasie może być ujemne prowadząc do $\dot{H} > 0$ (super-infacja). Dla $a \gg a_* \ \Rightarrow \ D \approx 1$ prowadząc do $\dot{H} < 0$ (deceleracja). Dla $a \ll a_*$ dobrym przybliżeniem jest

$$D(q) \approx \left(\frac{3}{1+l}\right)^{3/(2-2l)} \left(\frac{a}{a_*}\right)^{3(2-l)/(1-l)}$$

Wykorzystując to wyrażenie liczymy dynamikę w obszarze semiklasycznym. Teraz

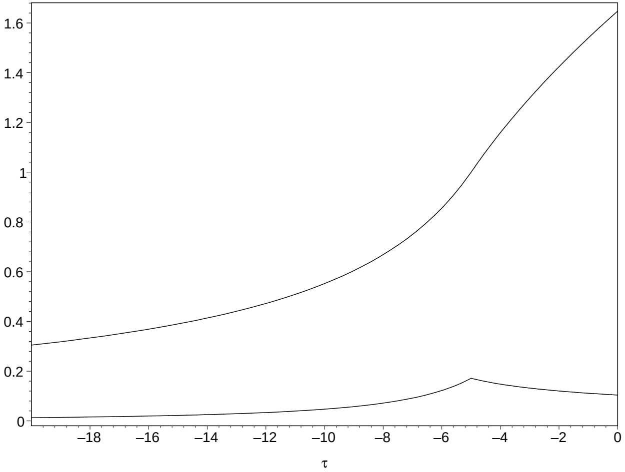
$$\frac{\dot{D}}{HD} = \frac{3(2-l)}{1-l} > 6$$

Warunki zszycia dla rozwiązań:

$$\begin{aligned} a_1(-\tau_0) &= a_2(-\tau_0) \\ a_1'(-\tau_0) &= a_2'(-\tau_0). \end{aligned}$$

Rozwiazania:

$$\begin{aligned} a_1(\tau) &= a_* \left(-\frac{\tau}{\tau_0}\right)^{-2\frac{1-l}{2+l}} \quad \text{dla} \quad \tau < -\tau_0 \\ a_2(\tau) &= a_* \sqrt{4\frac{1-l}{2+l}\left(\frac{\tau_0+\tau}{\tau_0}\right)+1} \quad \text{dla} \quad \tau > -\tau_0 \quad . \end{aligned}$$


1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
–18
–16
–14
–12
–10
–8
–6
–4
–2
0
τ

Tensorowe zaburzenie h_{ij} do metryki FRW zapisujemy w postaci

$$ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

gzdie $|h_{ij}| \ll 1$. To zaburzenie odpowiada falom grawitacyjnym z dwoma stopniami swobody. To pozwala na dekompozycje

$$h_{ij} = h_+ e^+_{ij} + h_\times e^\times_{ij}$$

gdzie

$$e^+_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{oraz} \quad e^\times_{ij} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Zaburzone równanie Einsteina, w przestrzeni Fouriera, ma postać

$$h_j^{i''} + 2\mathcal{H} h_j^{i'} + k^2 h_j^i = 0$$

Wporwadzając zmienną $\mu = ah$ możemy to równanie zapisać w postaci

$$\mu''_{\vec{k}} + \left[k^2 - \frac{a''}{a} \right] \mu_{\vec{k}} = 0.$$

Dla super-inflacyjnej fazy dostajemy

$$\mu_k = \frac{\mathcal{N}}{\sqrt{2k}} \sqrt{-k\tau} H^{(1)}_{\beta+\frac{1}{2}}(-k\tau)$$

gdzie

$$\mathcal{N} = \sqrt{\frac{\pi}{2}} e^{i\pi(\nu+1/2)/2}$$

Normalizaja ze zgodności z $e^{-ik\tau}/\sqrt{2k}$ dla $k \ll 1$.

Dla superhoryzonatlnych modów

$$H_{\nu}^{(1)}(-k\tau) \simeq -\frac{i}{\pi}\Gamma(\nu)\left(-\frac{k\tau}{2}\right)^{-\nu}$$

Z definicji

$$\mathcal{P}_T(k) = \frac{4l_{pl}^2}{a^2\pi^2}k^3|\mu_k|^2$$

Widmo zaburzeń tensorowych

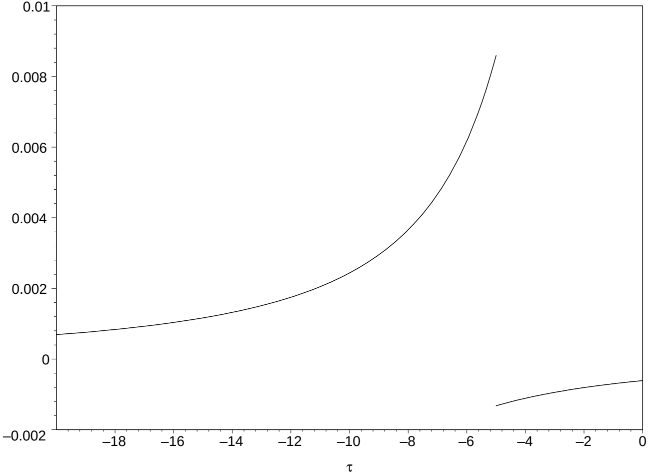
$$\mathcal{P}_T(k) = \mathcal{A}_T^2\left(\frac{k}{aH}\right)^{n_T}$$

gdzie

$$n_T = \frac{d\ln\mathcal{P}_T}{d\ln k} = \frac{6l}{2+l}$$

oraz

$$\mathcal{A}_T^2 = \frac{H^2}{m_p^2}\frac{2^{2\beta+1}}{\pi}\Gamma^2(\beta+1/2)$$



0.01
0.008
0.006
0.004
0.002
0
–0.002
–18
–16
–14
–12
–10
–8
–6
–4
–2
0
τ

$$\hat{\mu} = \frac{1}{2(2\pi)^{3/2}} \int d^3k \left[\hat{\mu}_{\vec{k}} e^{-i\vec{k}\cdot\vec{x}} + \hat{\mu}^{\dagger}_{\vec{k}} e^{i\vec{k}\cdot\vec{x}} \right]$$

$$\hat{\pi} = \frac{1}{2(2\pi)^{3/2}} \int d^3k \left[\hat{\pi}_{\vec{k}} e^{-i\vec{k}\cdot\vec{x}} + \hat{\pi}^{\dagger}_{\vec{k}} e^{i\vec{k}\cdot\vec{x}} \right]$$

gdzie $\pi = \mu'$, oraz

$$[\hat{\mu}(\vec{x},\tau), \hat{\pi}(\vec{x},\tau)] = i\delta^{(3)}(\vec{x}-\vec{y}).$$

Dla $a_1(\tau) = a_*(-\tau/\tau_1)^{-\beta}$ z $\beta = 2\frac{1-l}{2+l}$ mamy

$$\begin{aligned} \hat{\mu}_{\vec{k}}(\tau) &= \hat{a}_{\vec{k}} f_1(k,\tau) + \hat{a}^\dagger_{-\vec{k}} f_1^*(k,\tau) \quad \text{for} \quad \tau < -\tau_0 \\ \hat{\pi}_{\vec{k}}(\tau) &= \hat{a}_{\vec{k}} g_1(k,\tau) + \hat{a}^\dagger_{-\vec{k}} g_1^*(k,\tau) \quad \text{for} \quad \tau < -\tau_0 \end{aligned}$$

$$\begin{aligned} f_1(k,\tau) &= \frac{\mathcal{N}_1}{\sqrt{2k}} \sqrt{-k\tau} H_\nu^{(1)}(-k\tau) \\ g_1(k,\tau) &= -\mathcal{N}_1 \sqrt{\frac{k}{2}} \sqrt{-k\tau} \left[-H_{\nu+1}^{(1)}(-k\tau) + \frac{1+2\nu}{2(-k\tau)} H_\nu^{(1)}(-k\tau) \right] \end{aligned}$$

gdzie

$$\mathcal{N}_1 = \sqrt{\frac{\pi}{2}} e^{i\pi(\nu+1/2)/2} \quad \text{oraz} \quad \nu = \beta + \frac{1}{2}$$

Dla $a_2(\tau) = a_* \sqrt{4 \frac{1-l}{2+l} \left(\frac{\tau_0+\tau}{\tau_0} \right) + 1}$ mamy

$$
\begin{aligned}
\hat{\mu}_{\vec{k}}(\tau) &= \hat{b}_{\vec{k}} f_2(k,\tau) + \hat{b}^{\dagger}_{-\vec{k}} f_2^*(k,\tau) \quad \text{for} \quad \tau > -\tau_0 \\
\hat{\pi}_{\vec{k}}(\tau) &= \hat{b}_{\vec{k}} g_2(k,\tau) + \hat{b}^{\dagger}_{-\vec{k}} g_2^*(k,\tau) \quad \text{for} \quad \tau > -\tau_0
\end{aligned}
$$

$$
\begin{aligned}
f_2(k,\tau) &= \mathcal{N}_2 \sqrt{1 + 4 \frac{1-l}{2+l} \left(\frac{\tau_0+\tau}{\tau_0} \right)} H_0^{(2)}\left(k\tau + k\zeta\right) \exp\left(ik\zeta\right) \\
g_2(k,\tau) &= \frac{\mathcal{N}_2}{\tau_0} \left[\frac{H_0^{(2)}\left(k\tau + k\zeta\right)}{\sqrt{1 + 4 \frac{1-l}{2+l} \left(\frac{\tau_0+\tau}{\tau_0} \right)}} \frac{2(1-l)}{2+l} - k\tau_0 \sqrt{1 + 4 \frac{1-l}{2+l} \left(\frac{\tau_0+\tau}{\tau_0} \right)} H_1^{(2)}\left(k\tau + k\zeta\right) \right] \exp\left(ik\zeta\right)
\end{aligned}
$$

gdzie

$$
\mathcal{N}_2 = \frac{\sqrt{\pi}}{4} \sqrt{\tau_0} \sqrt{\frac{2+l}{1-l}} e^{-i\pi/4} \quad \text{oraz} \quad \zeta = \tau_0 \frac{3}{4} \frac{2-l}{1-l}
$$

Transformacja Bogoliubova-Valatina :

$$\begin{aligned}\hat{b}_{\vec{k}} &= B_+(k)\hat{a}_{\vec{k}} + B_-(k)^*\hat{a}^{\dagger}_{-\vec{k}} \\ \hat{b}^{\dagger}_{\vec{k}} &= B_+(k)^*\hat{a}^{\dagger}_{\vec{k}} + B_-(k)\hat{a}_{-\vec{k}}\end{aligned}$$

korzystając z wcześniejszych rozkładow na operatoty kreacji i anihilacji dostajemy

$$\begin{aligned}f_1(-\tau_0) &= B_+(k)f_2(-\tau_0) + B_-(k)f_2^*(-\tau_0) \\ g_1(-\tau_0) &= B_+(k)g_2(-\tau_0) + B_-(k)g_2^*(-\tau_0)\end{aligned}$$

Lub rozwiązując na współczyniki Bogoliubova

$$\begin{aligned}B_-(k) &= \frac{f_1(-\tau_0)g_2(-\tau_0) - g_1(-\tau_0)f_2(-\tau_0)}{f_2^*(-\tau_0)g_2(-\tau_0) - g_2^*(-\tau_0)f_2(-\tau_0)} \\ B_+(k) &= \frac{f_1(-\tau_0)g_2^*(-\tau_0) - g_1(-\tau_0)f_2^*(-\tau_0)}{f_2(-\tau_0)g_2^*(-\tau_0) - g_2(-\tau_0)f_2^*(-\tau_0)}\end{aligned}$$

Przejście między dwoma stanami próżni powoduje produkcję cząstek (grawiatonow)

$$\bar{n}_{\vec{k}} = \frac{1}{2}\langle 0|[\hat{b}^{\dagger}_{\vec{k}}\hat{b}_{\vec{k}} + \hat{b}^{\dagger}_{-\vec{k}}\hat{b}_{-\vec{k}}]|0\rangle = |B_{-}(k)|^2$$

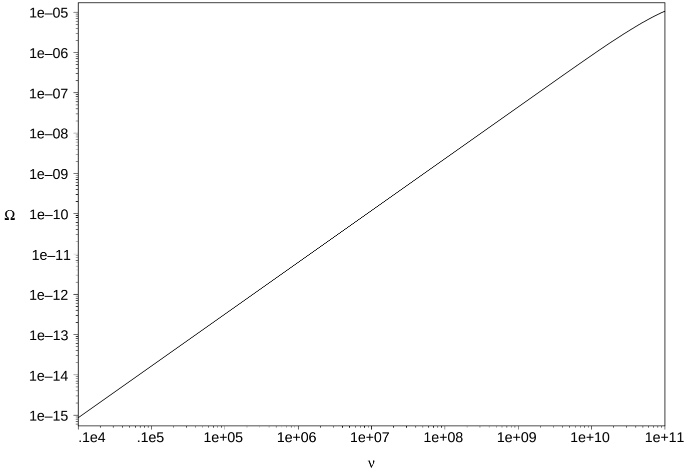
Gestość wyprodukowanych grawitonow wyliczamy z

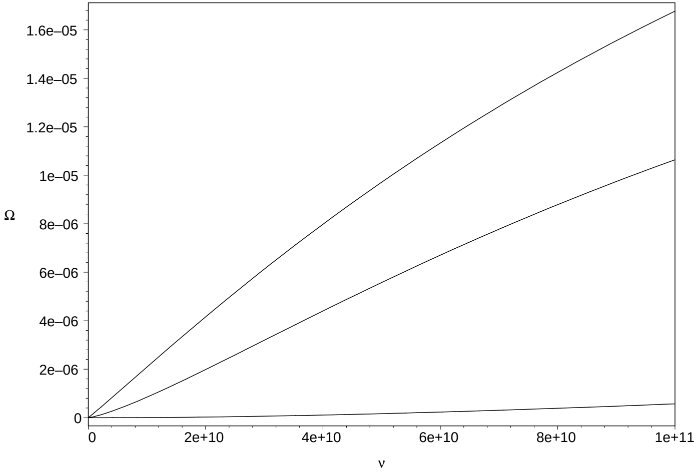
$$d\rho_{gw} = 2 \cdot \hbar\omega \frac{dk^3}{(2\pi)^3}\bar{n}_{\vec{k}}$$

Zazwyczaj gestość tła grawitonowego opisujemy parametrem

$$\Omega_{gw}(\nu,\tau) = \frac{1}{\rho_{\text{crit}}}\frac{d\rho_{GW}}{d\ln\nu}$$

Ograniczenie z obserwacji LIGO: $\Omega_{gw} < 6.5 \cdot 10^{-5}$.


1e–05
1e–06
1e–07
1e–08
1e–09
Ω
1e–10
1e–11
1e–12
1e–13
1e–14
1e–15
.1e4
.1e5
1e+05
1e+06
1e+07
1e+08
1e+09
1e+10
1e+11
ν


1.6e–05
1.4e–05
1.2e–05
1e–05
8e–06
6e–06
4e–06
2e–06
0
Ω
0
2e+10
4e+10
6e+10
8e+10
1e+11
ν