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Gravitational waves from the big bounce

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Abstract. In this paper we investigate gravitational wave production during the big bounce phase, inspired by loop quantum cosmology. We consider the influence of the holonomy corrections to the equation for tensor modes. We show that they act like additional effective graviton mass, suppressing gravitational wave creation. However, such effects can be treated perturbatively. We investigate a simplified model without holonomy corrections to the equation for modes and find its exact analytical solution. Assuming the form for matter $\rho \propto a^{-2}$ we calculate the full spectrum of the gravitational waves from the big bounce phase. The spectrum obtained decreases to zero for the low energy modes. On the basis of this observation we infer that this effect can lead to low cosmic microwave background (CMB) multipole suppression and gives a potential way for testing loop quantum cosmology models. We also consider a scenario with a post-bounce inflationary phase. The power spectrum obtained gives a qualitative explanation of the CMB spectra, including low multipole suppression.

Keywords: gravity waves/theory, quantum gravity phenomenology, physics of the early universe

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Contents

1. Introduction

Gravitational waves seems to be the best tool to use to explore the early universe. In particular they can be potentially used to verify quantum cosmological models. This idea is based on the fact that gravitational waves produced during the quantum epoch can survive frozen on super-horizon scales. Then after re-entering the horizon they can make an imprint on the CMB spectrum, which is observed today. From the empirical point of view, the most promising is the B spectrum of the CMB polarization. This spectrum has its source only in the tensor part of perturbation and when observed gives a direct method for investigating relic gravitational waves. It is expected that the PLANCK mission could give an opportunity for detecting B polarization [1]. However, such predictions are based on simple inflationary models.

One of the most promising approaches to quantizing gravity is that of loop quantum gravity (LQG) [2]. On the basis of this the theory of the quantum universe, loop quantum cosmology [3, 4], arose. This theory predicts that the initial singularity state is replaced by the quantum big bounce [5]. In this scenario a universe undergoes contraction and thereafter the quantum bounce evolves toward an expanding phase. During the bounce, the energy density reaches the maximal finite energy density ρ_c. A phenomenological description of the bounce phase can be obtained from the modified Friedmann equation

$$H^2 = \frac{\kappa}{3}\rho\left(1-\frac{\rho}{\rho_c}\right) \tag{1.1}$$

where $\kappa = 8\pi G$. It is worth mentioning that a similar equation appears also in the brane cosmologies [6].

Investigations of the perturbations in the cosmological models are crucial as regards large scale structure creation and exploration of the early universe. In the loop quantum cosmology this issue has been studied in [7]–[10]. Inhomogeneities in loop quantum cosmology are introduced by perturbations of the Ashtekar variables. However, since no clear connection between this theory and full loop quantum gravity exists, the meaning of these perturbations must to be considered carefully. In particular, we should keep it in mind that space below the Planck length becomes discrete and the notion of continuity breaks down. The effective description, with quantum correction, takes into account discreteness effects but keeps the notion of continuity. Recently some progress on this issue has been made in [11], where inhomogeneous cosmology was derived from the spin-foam model.

In the present paper we consider a particular kind of metric perturbation, the gravitational waves. The creation of gravitons in models inspired by the loop quantum cosmology was initially studied in [12, 13]. Then the equation for the tensor modes with holonomy and inverse volume corrections was derived in [14]. Recently an equation with holonomy effects has been applied to the inflationary phase [15].

In the present paper we investigate the creation of gravitational waves during the big bounce phase. To obtain analytical solutions of the model we assume a matter content in the form $\rho \propto a^{-2}$. In our considerations we take into account only holonomy effects (subsequently we neglect them in the analytical model). The inverse volume corrections exhibit a fiducial cell dependence and are not appropriate in models with a flat background [16]. The production of perturbations in the bouncing cosmologies has recently been studied in a different context in [17]–[20].

2. Tensor modes with holonomy corrections

The equation for tensor modes with holonomy corrections has been derived by Bojowald and Hossain [14]. In the source free case these equation take the form

$$\frac{\mathrm{d}^2}{\mathrm{d}\tau^2}h_i + 2\left(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma}\right)\frac{\mathrm{d}}{\mathrm{d}\tau}h_i - \nabla^2 h_i + T_Q h_i = 0 \tag{2.1}$$

where $i = \oplus, \otimes$ indicates the polarization state and

$$T_Q = -2\left(\frac{\bar{p}}{\bar{\mu}}\frac{\partial\bar{\mu}}{\partial\bar{p}}\right)\bar{\mu}^2\gamma^2\left[\frac{\sin\left(\bar{\mu}\gamma\bar{k}\right)}{\bar{\mu}\gamma}\right]^4. \tag{2.2}$$

Here $\bar{p} = a^2$ and $\bar{k} = \bar{p}'/2\bar{p}$. The Hamilton equation for the variable $\bar{p}$ takes the form [14]

$$\bar{p}' = 2\bar{p}\left(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma}\right), \tag{2.3}$$

which leads to

$$\left(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma}\right) = \frac{a'}{a}. \tag{2.4}$$

The above equality indicates that the friction term in equation (2.1) holds its classical form. The effect of the holonomies is the additional term T_Q correcting the classical equation for tensor modes. This factor acts like an additional effective graviton mass.

To define the correction T_Q we need to specify a function $\bar{\mu}$. In general there is some freedom as regards the choice of this function in the power law form. However, it has been recently shown that for the flat FRW models the only consistent choice is [21]

$$\bar{\mu} = \sqrt{\frac{\Delta}{\bar{p}}} \tag{2.5}$$

where $\Delta = 2\sqrt{3}\pi\gamma l_{\mathrm{Pl}}^2$, which is called a $\bar{\mu}$-scheme.

Introducing a new variable

$$u = \frac{ah_{\oplus}}{\sqrt{16\pi G}} = \frac{ah_{\otimes}}{\sqrt{16\pi G}} \tag{2.6}$$

and taking the Fourier transform

$$u(\tau, \mathbf{x}) = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} u(\tau, \mathbf{k}) \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}} \tag{2.7}$$

we can rewrite equation (2.1) in the form

$$\frac{\mathrm{d}^2}{\mathrm{d}\tau^2} u(\tau, \mathbf{k}) + [k^2 + m_{\mathrm{eff}}^2] u(\tau, \mathbf{k}) = 0 \tag{2.8}$$

where $k^2 = \mathbf{k}\cdot\mathbf{k}$ and

$$m_{\mathrm{eff}}^2 = T_Q - \frac{a''}{a}. \tag{2.9}$$

To calculate this function we must to specify the background dynamics. We consider a model with a free scalar field. In this case the evolution of the parameter $\bar{p}$ takes the form [22]

$$\bar{p} = \left(A + Bt^2\right)^{1/3} \tag{2.10}$$

where

$$A = \tfrac{1}{6}\kappa\pi_\phi^2\gamma^2\Delta \qquad \text{and} \qquad B = \tfrac{3}{2}\kappa\pi_\phi^2. \tag{2.11}$$

On the basis of definition (2.9) we calculate

$$m_{\mathrm{eff}}^2 = \frac{\kappa^2\pi_\phi^4}{4} \frac{\left(t^2 - \frac{2}{9}\gamma^2\Delta\right)}{\left(A + Bt^2\right)^{5/3}}. \tag{2.12}$$

In the case $T_Q = 0$ we obtain

$$m_{\mathrm{eff}}^2(T_Q = 0) = \frac{\kappa^2\pi_\phi^4}{4} \frac{\left(t^2 - \frac{1}{3}\gamma^2\Delta\right)}{\left(A + Bt^2\right)^{5/3}}. \tag{2.13}$$

In figure 1 we show the evolution of the effective masses m_{eff}^2 and $m_{\mathrm{eff}}^2(T_Q = 0)$. The values of these functions are crucial from the point of view of gravitational wave production. Generally, the more negative this function, the greater the gravitational wave production. We see that the presence of correction T_Q leads to suppression of the graviton production. However, this effect is relatively weak.

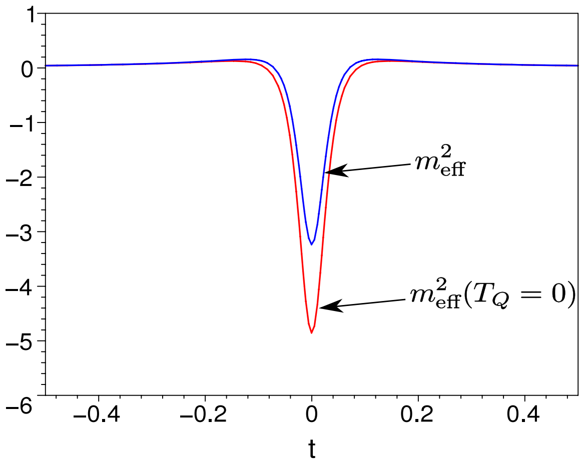

Figure 1. Evolution of the effective masses m^2_{eff} and $m^2_{\text{eff}}(T_Q = 0)$.

For the bouncing universe considered we can write a closed system of equations:

$$\frac{\mathrm{d}u}{\mathrm{d}\tau} = \pi_u, \tag{2.14}$$

$$\frac{\mathrm{d}\pi_u}{\mathrm{d}\tau} = -\left[k^2 + \frac{\kappa^2\pi_\phi^4}{4}\frac{\left(t^2 - \frac{2}{9}\gamma^2\Delta\right)}{(A+Bt^2)^{5/3}}\right]u, \tag{2.15}$$

$$\frac{\mathrm{d}t}{\mathrm{d}\tau} = \left(A + Bt^2\right)^{1/6}, \tag{2.16}$$

to describe the evolution of the gravitational waves. This system contains the holonomy correction to the background dynamics as well as that to the correction to the equation for tensor modes. Numerical solutions of this system can fully determine the classical evolution of the gravitational waves during the big bounce phase.

In figure 2 we show typical solutions for the function u and h_i for the vacuum initial condition $u \sim 1/\sqrt{k}$. It is transparent that the tensor modes are amplified during the bounce phase.

In the next section we aim to determine qualitative and quantitative properties of the gravitons produced during the bounce phase.

3. Toy model

The goal of this section is to give an introduction to the process of particle production during the bounce phase. We show a simple, fully analytical model of the gravitational wave creation during the bounce. The model considered does not contain any quantum holonomy corrections to the mode equation. This simplification can be however justified. Namely, as was shown in the previous section, the holonomy correction T_Q leads only to perturbative modifications.

The model considered is based on two assumptions:

- we neglect quantum holonomy corrections T_Q in the mode equation,

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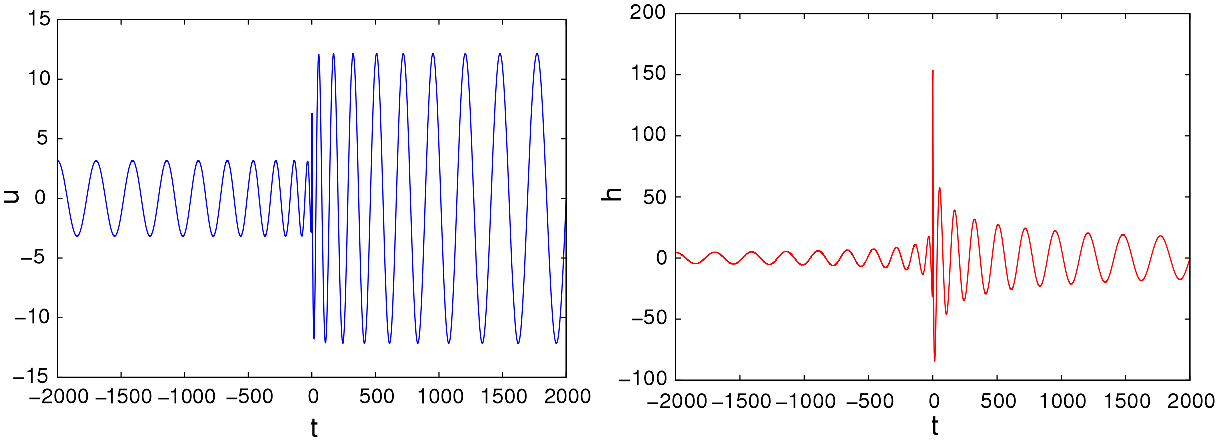

Figure 2. Left: evolution of the field u. Right: amplification of the tensor modes h during the bounce.

- we assume matter content in the form

$$\rho = \frac{\rho_c}{a^2}. \tag{3.1}$$

3.1. Background dynamics

Plugging condition (3.1) into the modified Friedmann equation (1.1) we find the solution

$$a(t) = \sqrt{1 + (t/t_0)^2} \tag{3.2}$$

where

$$t_0^2 = \frac{3}{8\pi G \rho_c}. \tag{3.3}$$

For the further calculations it will be useful to express this solution in terms of the conformal time. Performing the transformation $d\tau = dt/a$ we find

$$a(\tau) = \cosh(\tau/t_0). \tag{3.4}$$

3.2. Mode functions

Equation (2.8) without the term T_Q takes the form

$$\frac{d^2}{d\tau^2} u(\tau, \mathbf{k}) + \left[k^2 - \frac{a''}{a} \right] u(\tau, \mathbf{k}) = 0. \tag{3.5}$$

This equation can be obtained from the action

$$S_t = \frac{1}{2} \int d\tau \, d^3\mathbf{x} \, [u'^2 - \delta^{ij} \partial_i u \, \partial_j u - m_{\text{eff}}^2 u^2] \tag{3.6}$$

where

$$m_{\text{eff}}^2 = -\frac{a''}{a}. \tag{3.7}$$

Canonical momenta conjugate to the variable u are obtained from

$$\pi(\tau, \mathbf{x}) = \frac{\delta S_{\mathrm{t}}}{\delta u'} = u'. \tag{3.8}$$

The next step is to quantize this theory. We perform the canonical quantization $(u, \pi) \to (\hat{u}, \hat{\pi})$ introducing relations of the commutations $[\hat{u}(\mathbf{x}, \tau), \hat{\pi}(\mathbf{y}, \tau)] = \mathrm{i}\delta^{(3)}(\mathbf{x} - \mathbf{y})$ and $[\hat{u}(\mathbf{x}, \tau), \hat{u}(\mathbf{y}, \tau)] = [\hat{\pi}(\mathbf{x}, \tau), \hat{\pi}(\mathbf{y}, \tau)] = 0$. The operators $\hat{u}, \hat{\pi}$ can be decomposed for the Fourier modes:

$$\hat{u}(\tau, \mathbf{x}) = \frac{1}{2(2\pi)^{3/2}} \int \mathrm{d}^3\mathbf{k} \left[\hat{u}_{\mathbf{k}}(\tau)\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}} + \hat{u}^{\dagger}_{\mathbf{k}}(\tau)\mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{x}}\right],$$

$$\hat{\pi}(\tau, \mathbf{x}) = \frac{1}{2(2\pi)^{3/2}} \int \mathrm{d}^3\mathbf{k} \left[\hat{\pi}_{\mathbf{k}}(\tau)\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}} + \hat{\pi}^{\dagger}_{\mathbf{k}}(\tau)\mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{x}}\right]$$

where

$$\hat{u}_{\mathbf{k}}(\tau) = \hat{a}_{\mathbf{k}} f(k, \tau) + \hat{a}^{\dagger}_{-\mathbf{k}} f^*(k, \tau), \tag{3.9}$$

$$\hat{\pi}_{\mathbf{k}}(\tau) = \hat{a}_{\mathbf{k}} g(k, \tau) + \hat{a}^{\dagger}_{-\mathbf{k}} g^*(k, \tau), \tag{3.10}$$

and $f(k, \tau)' = g(k, \tau)$.

The equation for the mode function takes the form

$$\frac{\mathrm{d}^2}{\mathrm{d}\tau^2} f(k, \tau) + \left[k^2 + m^2_{\mathrm{eff}}\right] f(k, \tau) = 0 \tag{3.11}$$

where

$$m^2_{\mathrm{eff}} = -\frac{a''}{a} = -\frac{1}{t_0^2} = -k_0^2. \tag{3.12}$$

With the use of the definition

$$t_0^2 = \frac{3}{8\pi G \rho_{\mathrm{c}}} \qquad \text{and} \qquad \rho_{\mathrm{c}} = \frac{\sqrt{3}}{16\pi^2\gamma^3 l^4_{\mathrm{Pl}}} \tag{3.13}$$

and assuming $\gamma = \gamma_M = 0.2375$ [23] we obtain

$$k_0 \simeq \frac{2.62}{l_{\mathrm{Pl}}}. \tag{3.14}$$

Now we can find solutions of equation (3.11). We consider two cases, $k^2 > k_0^2$ and $k^2 < k_0^2$.

- Case $k^2 > k_0^2$

  The solution of equation (3.11) takes the form

$$f(k, \tau) = A\mathrm{e}^{-\mathrm{i}\Omega\tau} + B\mathrm{e}^{\mathrm{i}\Omega\tau} \tag{3.15}$$

  where $A, B \in \mathbb{C}$ and

$$\Omega = \sqrt{k^2 - k_0^2}. \tag{3.16}$$

  Performing the normalization, with the use of the Wronskian condition, we obtain

$$|A|^2 - |B|^2 = \frac{1}{2\Omega}. \tag{3.17}$$

We choose advanced modes taking

$$A = \frac{1}{\sqrt{2\Omega}} \qquad \text{and} \qquad B = 0, \tag{3.18}$$

which gives

$$f(k,\tau) = \frac{1}{\sqrt{2\Omega}} \mathrm{e}^{-\mathrm{i}\Omega\tau}, \tag{3.19}$$

$$g(k,\tau) = f'(k,\tau) = -\mathrm{i}\sqrt{\frac{\Omega}{2}} \mathrm{e}^{-\mathrm{i}\Omega\tau}. \tag{3.20}$$

- Case $k^2 < k_0^2$
The solution of equation (3.11) takes the form

$$f(k,\tau) = A\mathrm{e}^{-\bar{\Omega}\tau} + B\mathrm{e}^{\bar{\Omega}\tau} \tag{3.21}$$

where $A, B \in \mathbb{C}$ and

$$\bar{\Omega} = \sqrt{k_0^2 - k^2}. \tag{3.22}$$

Performing the normalization, with the use of the Wronskian condition, we obtain

$$BA^* - AB^* = -\frac{\mathrm{i}}{2\bar{\Omega}}, \tag{3.23}$$

which is fulfilled by

$$A = \frac{\mathrm{i}}{2\sqrt{\bar{\Omega}}} \qquad \text{and} \qquad B = \frac{1}{2\sqrt{\bar{\Omega}}}. \tag{3.24}$$

In this case we obtain

$$f(k,\tau) = \frac{1}{2\sqrt{\bar{\Omega}}} \left[\mathrm{e}^{\bar{\Omega}\tau} + \mathrm{i}\mathrm{e}^{-\bar{\Omega}\tau} \right], \tag{3.25}$$

$$g(k,\tau) = f'(k,\tau) = \frac{\sqrt{\bar{\Omega}}}{2} \left[\mathrm{e}^{\bar{\Omega}\tau} - \mathrm{i}\mathrm{e}^{-\bar{\Omega}\tau} \right]. \tag{3.26}$$

3.3. Power spectrum

The correlation function for the tensor modes takes the form

$$\begin{aligned} \langle 0|\hat{h}^a_b(\vec{x},\tau)\hat{h}^b_a(\vec{y},\tau)|0\rangle &= 4\frac{16\pi G}{a^2}\int \frac{\mathrm{d}^3k}{(2\pi)^3}|f(k,\tau)|^2\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{r}} \\ &= \int \frac{\mathrm{d}k}{k}\mathcal{P}_T(k,\tau)\frac{\sin kr}{kr} \end{aligned} \tag{3.27}$$

where we have defined the power spectrum

$$\mathcal{P}_T(k,\tau) = \frac{64\pi G}{a^2}\frac{k^3}{2\pi^2}|f(k,\tau)|^2. \tag{3.28}$$

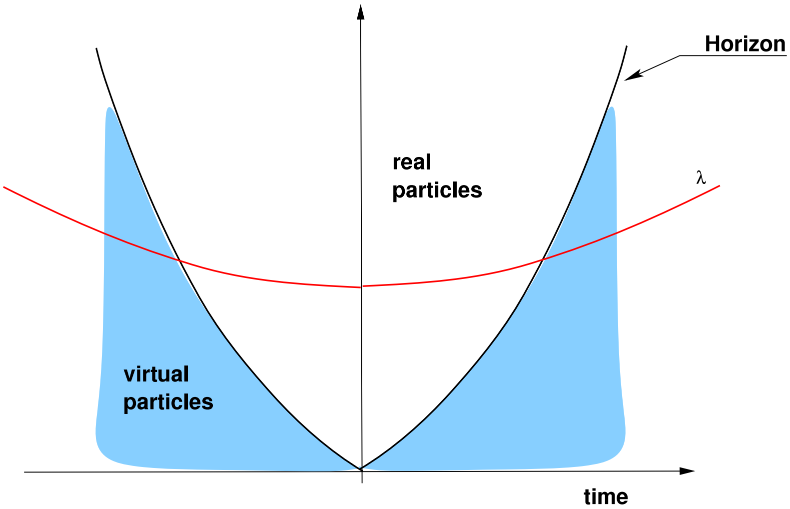

Figure 3. The horizon in the bouncing cosmologies.

Our goal now is to determine this spectrum on the Hubble (horizon) scales. The evolution of the Hubble radius during the bouncing phase considered takes the form

$$ds^2 = 0 \rightarrow R_{\rm H} = \pm a(\tau)\int_0^{\tau} {\rm d}\tau' = \pm a(\tau)\tau \tag{3.29}$$

where $+$ denotes *expansion* and $-$ denotes *contraction*. We can also express it in terms of the coordinate time t, obtaining

$$R_{\rm H} = \pm a(t)\int_0^t \frac{{\rm d}t'}{a(t')} = \pm t_0\sqrt{1+(t/t_0)^2}{\rm arcsinh}(t/t_0). \tag{3.30}$$

This function decreases to zero in the pre-big bang phase and increases in the post-big bang phase. In figure 3 we show the evolution of the horizon for the typical bouncing cosmologies. We show also the evolution of the arbitrary length scale $\lambda \propto a$. We see that during the bouncing phase, all length scales finally cross the horizon. Quantum fluctuations of the length $\lambda < R_{\rm H}$ behave like virtual particles, when fluctuations of the length $\lambda > R_{\rm H}$ become classical excitations.

The condition $\lambda = R_{\rm H}$ indicates

$$k = \frac{2\pi a}{R_{\rm H}} = \frac{2\pi a}{\pm\tau a} = \frac{2\pi}{\pm\tau}. \tag{3.31}$$

Plugging it into definition (3.28) and using the mode functions (3.19), (3.25) we determine the tensor power spectrum on the horizon scales:

$$\mathcal{P}_T(k) = \begin{cases} \dfrac{16}{\pi}\left(\dfrac{k_0}{m_{\rm Pl}}\right)^2\left(\dfrac{k}{k_0}\right)^3 \dfrac{1}{\sqrt{(k/k_0)^2-1}\cosh^2\left(2\pi/(k/k_0)\right)} & \text{for } k > k_0 \\ \dfrac{16}{\pi}\left(\dfrac{k_0}{m_{\rm Pl}}\right)^2\left(\dfrac{k}{k_0}\right)^3 \dfrac{\cosh\left[4\pi\sqrt{(k_0/k)^2-1}\right]}{\sqrt{1-(k/k_0)^2}\cosh^2\left(2\pi/(k/k_0)\right)} & \text{for } k < k_0. \end{cases} \tag{3.32}$$

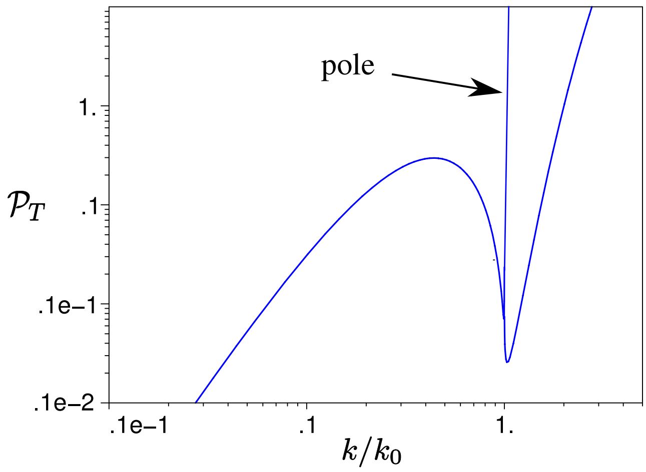

Figure 4. Spectrum of the tensor perturbations on the horizon.

We plot this spectrum in figure 4. We see that for $k < k_0$ the spectrum is characterized by the bump. For $k \to 0$ the spectrum is decreasing to zero. However, for $k \to k_0$ the spectrum has a pole $\mathcal{P}_T \to \infty$. This pole is however not physical. To show this we consider the correlation function

$$\langle 0|\hat{h}^a_b(\vec{x},\tau)\hat{h}^b_a(\vec{x},\tau)|0\rangle = \frac{32G}{\pi a^2}\int_0^\infty \mathrm{d}k\, k^2 |f(k,\tau)|^2 \tag{3.33}$$

where

$$|f(k,\tau)|^2 = \Theta(k-k_0)\frac{1}{2\sqrt{k^2-k_0^2}} + \Theta(k_0-k)\frac{1}{2\sqrt{k_0^2-k^2}}\cosh\left[2\sqrt{k_0^2-k^2}\tau\right].$$

The correlation function is a physical quantity and can indicate which features of the spectrum $\mathcal{P}_T$ are real divergences. As we can see, near the pole $k = k_*$ the correlation function is determined by the integrals

$$I_1 = \int \mathrm{d}k\, \frac{k^2}{\sqrt{k^2-k_0^2}} = \left\{x = \frac{k}{k_0}\right\} = \frac{k_0^2}{2}\left[x\sqrt{x^2-1} - \ln\left|x+\sqrt{x^2-1}\right|\right] \tag{3.34}$$

and

$$I_2 = \int \mathrm{d}k \frac{k^2}{\sqrt{k_0^2-k^2}} = \left\{x = \frac{k}{k_0}\right\} = \frac{k_0^2}{2}\left[-x\sqrt{1-x^2} - \mathrm{arcsin}x\right]. \tag{3.35}$$

Both of these are finite for $x = 1$ and give a finite contribution to the definition of the correlation function.

For $k \to \infty$ the power spectrum exhibit a UV divergence. In fact, the region $k > k_0$ traces the Planck scales and a different approach should be used to determine properties of the quantum fluctuations. Another important issue is indicating which of the modes survive frozen above the Hubble radius. In fact, it is not all of them, but only these with $k < k_0$. This can be seen from the asymptotic solutions of equation (3.5). Namely, in the regime $k^2 \ll |a''/a| = k_0^2$ we have the solution

$$h_i \simeq A_k + B_k \int^{\tau} \frac{\mathrm{d}x}{a^2(x)}. \tag{3.36}$$

Here we have a constant contribution A_k which freezes the amplitude of the gravitational waves. In this regime the spectrum does not change during the evolution above the horizon and will be the same on re-entering the horizon. In the regime $k^2 \gg |a''/a| = k_0^2$ we have decaying solutions

$$h_i \simeq \frac{\mathrm{e}^{\pm \mathrm{i}k\tau}}{a}. \tag{3.37}$$

These modes do not lead to classical fluctuations for re-entering the horizon.

Summing up, the spectrum $\mathcal{P}_T$ on the second branch of the horizon is free from the UV divergent part for $k > k_0$. The only contribution to this spectrum comes from the bump for $k < k_0$.

3.4. Parameter Ω_{gw}

To describe the spectrum of gravitons it is common to use the parameter

$$\Omega_{\mathrm{gw}}(\nu) = \frac{\nu}{\rho_*}\frac{\mathrm{d}\rho_{\mathrm{gw}}}{\mathrm{d}\nu} \tag{3.38}$$

where ρ_{gw} is the energy density of gravitational waves and ρ_* is present critical energy density. Our goal in this section is to calculate the function $\Omega_{\mathrm{gw}}(\nu)$ for the gravitons produced during the big bounce phase.

We consider the creation of the gravitons during the transition from some initial state to some final state. The initial vacuum state $|0_{\mathrm{in}}\rangle$ is determined by $\hat{a}_{\mathbf{k}}|0_{\mathrm{in}}\rangle = 0$, where $\hat{a}_{\mathbf{k}}$ is the initial annihilation operator for τ_i. The relation between annihilation and creation operators for the initial and final states is given by the Bogoliubov transformation

$$\hat{b}_{\mathbf{k}} = B_+(k)\hat{a}_{\mathbf{k}} + B_-(k)^*\hat{a}^{\dagger}_{-\mathbf{k}}, \tag{3.39}$$

$$\hat{b}^{\dagger}_{\mathbf{k}} = B_+(k)^*\hat{a}^{\dagger}_{\mathbf{k}} + B_-(k)\hat{a}_{-\mathbf{k}} \tag{3.40}$$

where $|B_+|^2 - |B_-|^2 = 1$. Because we are working in the Heisenberg description the vacuum state does not change during the evolution. It results that $\hat{b}_{\mathbf{k}}|0_{\mathrm{in}}\rangle = B_-(k)^*\hat{a}^{\dagger}_{-\mathbf{k}}|0_{\mathrm{in}}\rangle$ is different from zero when $B_-(k)^*$ is a non-zero function. This means that in the final state the graviton field considered is no longer in the vacuum state without particles. The number of particles produced in the final state is given by

$$\bar{n}_{\mathbf{k}} = \tfrac{1}{2}\langle 0_{\mathrm{in}}| \left[\hat{b}^{\dagger}_{\mathbf{k}}\hat{b}_{\mathbf{k}} + \hat{b}^{\dagger}_{-\mathbf{k}}\hat{b}_{-\mathbf{k}}\right] |0_{\mathrm{in}}\rangle = |B_-(k)|^2. \tag{3.41}$$

The energy density of gravitons is given by

$$\mathrm{d}\rho_{\mathrm{gw}} = 2\hbar\omega \cdot \frac{4\pi\omega^2\,\mathrm{d}\omega}{(2\pi c)^3} \cdot |B_-(k)|^2 \tag{3.42}$$

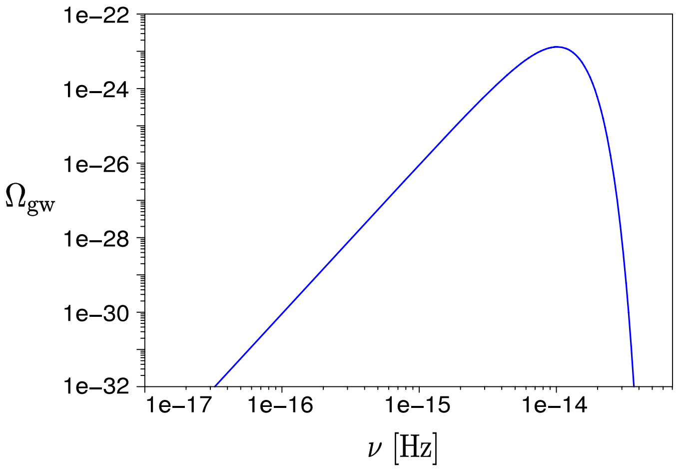

Figure 5. Function $\Omega_{\rm gw}(\nu)$ for the $\tau_{\rm i} = -20 l_{\rm Pl}$ and $\tau_{\rm f} = 20 l_{\rm Pl}$.

where we used the definition (3.41). The expression for the parameter $\Omega_{\rm gw}$ defined by (3.38) now takes the form

$$\Omega_{\rm gw}(\nu) = \begin{cases} 0 & \text{for } k > k_0 \\ \Omega_0 \nu^4 \sinh^2 \left[\sqrt{1-(k/k_0)^2} (\tau_{\rm i} - \tau_{\rm f}) k_0 \right] & \text{for } k \leq k_0 \end{cases} \tag{3.43}$$

where

$$k = 2\pi\nu \frac{a_0}{a_{\rm f}} \tag{3.44}$$

and

$$\Omega_0 = \frac{\hbar c}{c^4} \frac{16\pi^2}{\rho_*} = 3.66 h_0^{-2} \times 10^{-49}\ [{\rm Hz}^{-4}]. \tag{3.45}$$

For the model with the inflationary phase we have $(a_0/a_{\rm f}) \simeq 10^{56}$. Another value which must be specified is the duration of the bounce. We assume that $\tau_{\rm i} = -20 l_{\rm Pl}$ and $\tau_{\rm f} = 20 l_{\rm Pl}$.

In figure 5 we show the function $\Omega_{\rm gw}(\nu)$ for the set-up considered. The spectrum is characterized by the bump with the maximal frequency

$$\nu_{\rm max} \simeq 8 \times 10^{-14}\ [{\rm Hz}]. \tag{3.46}$$

The result presented supports our previous consideration. Namely, there is no production of gravitons with frequencies above some $\nu_{\rm max}$. The only contribution to the spectrum comes from the bump.

4. Tensor modes from the pre-bounce phase

The bouncing solution (2.10) in the limit $t \to \pm\infty$ gives

$$a(t) \propto |t|^{1/3} \propto |\tau|^{1/2}. \tag{4.1}$$

In this regime the correction T_Q vanishes and the expression for the effective mass simplifies to

$$m^2_{\rm eff} = -\frac{a''}{a} = \frac{1}{4}\frac{1}{\tau^2}. \tag{4.2}$$

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The equation for the mode functions takes the form

$$\frac{d^2}{d\tau^2} f(k,\tau) + \left[k^2 + \frac{1}{4}\frac{1}{\tau^2}\right] f(k,\tau) = 0. \tag{4.3}$$

The normalized solution of this equation has the form

$$f(k,\tau) = \frac{\mathcal{N}}{\sqrt{2k}}\sqrt{-\tau k}H_0^{(1)}(-\tau k) \tag{4.4}$$

where

$$\mathcal{N} = \sqrt{\frac{\pi}{2}}e^{i\pi/4}. \tag{4.5}$$

With the use of definitions (3.28) and (4.4) we obtain the power spectrum for the pre-bounce phase in the form

$$\mathcal{P}_T(k) = \sqrt{\frac{12}{\pi}}|H_0^{(1)}(2\pi)|^2 \left(\frac{k}{k_\#}\right)^3 \qquad \text{for } k \to 0 \tag{4.6}$$

where $k_\#$ is some constant. This general result has been known since the end of the 1970s [24].

It is important to note that this part of the spectrum does not depend on the quantum gravitational effects. This is in opposite to the predictions from the inflationary models, where low energy modes come from a high energy region. Here, low energy modes are produced in the low energy pre-big bang state. So, the present largest scale structures have their origin in the semi-classical pre-big bang phase rather than in the deep quantum regime.

5. Suppressing low CMB multipoles with a bounce + inflation scenario

In this section we consider a scenario with an inflationary phase taking place after the bounce. In figure 6 we show the resulting evolution of the horizon. We see that the horizon is firstly crossed by modes of length $\lambda > \lambda_*$. These modes live frozen on the super-horizon scales and re-enter the horizon in the post-big bang phase. Additionally they are not sensitive to the quantum gravitational effects close to the bounce. As was shown in the previous section, they lead to a power spectrum of the form

$$\mathcal{P}_T(k) \propto k^3 \qquad \text{for } k \ll k_*. \tag{5.1}$$

These fluctuations can lead directly to CMB fluctuations on large angular scales. Modes of the length $\lambda < \lambda_*$ also cross the horizon in the pre-big bang phase, but re-enter shortly after the bounce. They decay shortly after this and become quantum fluctuations. Then, during the inflationary phase they cross the horizon again. The spectrum of these fluctuations will be nearly flat, as predicted by the inflationary models:

$$\mathcal{P}_T(k) = \mathcal{A} \qquad \text{for } k \gg k_*. \tag{5.2}$$

The loop holonomy corrections to the inflationary spectrum were studied in [25, 26].

Finally, we should obtain the spectrum of the tensor perturbations in the form shown in figure 7.

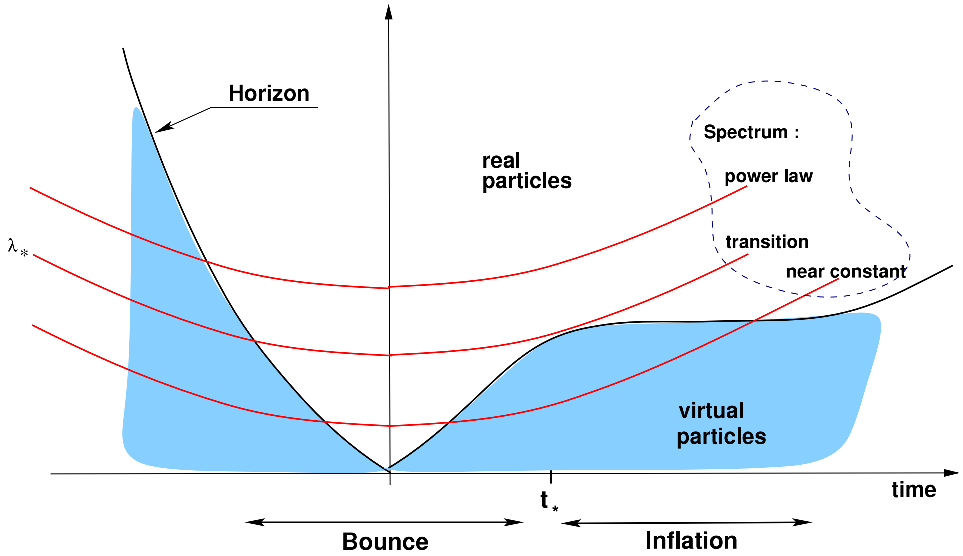

Figure 6. Evolution of the horizon in the bounce + inflation model.

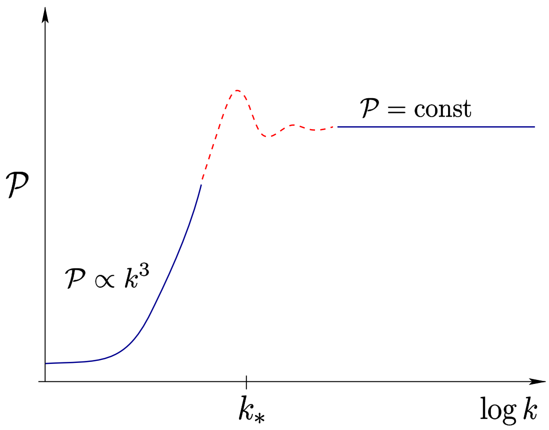

Figure 7. The schematic picture of the power spectrum expected for the bounce + inflation model.

To the describe the power spectrum in a continuous way we propose the phenomenological formula

$$\mathcal{P}(k) = \mathcal{A}\left(\frac{k}{k_*}\right)^3 \frac{1}{1+(k/k_*)^3},$$

which interpolates between the two regimes considered. We plot this function in figure 8.

The spectrum, in the same form, should also be obtained for the scalar modes of fluctuation. In this context similar models were studied in [27, 28]. They have shown that a transitional regime in the spectrum can have the form of oscillations, as was shown in figure 7. However, in the cited papers, evolution was obtained form the specific dynamics of a single inflationary field. Namely, no quantum gravitational correction was used to obtain the bounce phase.

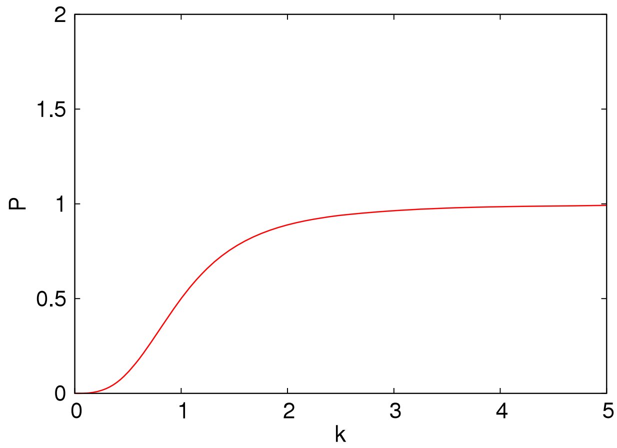

Figure 8. Phenomenological spectrum for the bounce + inflation model with $\mathcal{A} = 1$ and $k_* = 1$.

An interesting property of the spectra obtained is the damping of the low multipoles in the CMB spectra. Such an effect is in fact observed [29]. This gives potentially the possibility of testing bouncing cosmologies. If the bounce is present we should observe low multipole suppression. However, this effect can also have other origins and at the present stage of observations it is not possible to infer that it is truly a remnant of the big bounce.

The important feature of the scenario presented is that none of the primordial perturbations produced in the deep quantum gravitational regime give seeds for structure formation. All perturbations come from either the pre-big bang semi-classical phase or from the post-big bang inflationary phase. It is therefore hard to investigate observationally the deep quantum regime. However, we can potentially observe a classical pre-big bounce branch, which is the result of the quantum gravitational effects. As was shown in [30] the bounce + inflation model can be constructed in the framework of loop quantum cosmology (LQC). In the model with a power law potential the inflationary phase is an attractor of dynamics.

6. Summary

The present understanding of perturbations in LQC is at a preliminary stage. This is partially due to the fact that the connection with the full theory (LQG) is still not clear. However, a semi-classical effective approach, which is currently available, may give us some hints that are especially interesting as regards observations.

In this paper we have considered the gravitational wave creation during the big bounce phase, inspired by loop quantum cosmology. We have studied effects of the holonomy corrections to the equation for tensor modes. We have shown that they lead to the suppression of graviton production. However, this effect is not dominant and can be treated perturbatively. In fact, to obtain qualitative results it is justified to neglect this contribution. On the basis of the above studies we have solved the simplified model of graviton production during the bounce phase. We have neglected the holonomy correction in the equation for tensor modes and assumed the matter content $\rho \propto a^{-2}$. For this set-up we have derived the power spectrum $\mathcal{P}_T$ and the parameter Ω_{gw}. The spectrum obtained for the bounce has the form of a bump. It decreases to zero for energies tending to zero and for some high energy scale. So there is no contribution from the trans-Planckian regime and a natural cut-off appears. This seems to be a typical property of the bouncing cosmologies. However, such a cut-off does not appear in studies of the perturbations in the bouncing string cosmologies [17]. The low energy behaviour is still common. Namely, the spectrum is strongly suppressed on large scales. This is in fact a common feature of all models with a contracting phase (see [18, 19, 24, 27, 28]). In section 5 we have investigated the spectrum for the bounce + inflation model. In this case the spectrum decreases to zero for energies tending to zero and becomes nearly flat for high energies. Similar behaviour is also expected for the scalar perturbations. This was already shown in [27, 28], where contracting and inflationary solutions were joined. In those papers the approximation $z''/z \simeq a''/a$ for the variable $z \equiv a\phi'/\mathcal{H}$ was used. In such a case the equation for the Mukhanov variable takes the form (3.5). Therefore these calculations are like those in the gravitational wave case. The results of section 5 are in qualitative agreement with those in [27, 28]. Here an intuitive explanation of these results was given. The general conclusion is that low energy behaviour is common for all models with a contracting phase. However the high energy behaviour depends on the post-bounce evolution and details of the model.

The perturbations from the pre-big bang phase make a direct imprint in the large scale structures. In particular, they can lead to the suppression of the low CMB multipoles. It is worth mentioning that a similar prediction was achieved from the studies of super-inflation in loop quantum cosmology [31]. However the source of the suppression was quite different there; namely, the suppression was produced when the inflationary field reached its maximal value before finally slowly rolling down. In this model the inflationary field was driven up the potential well thanks to inverse volume corrections. From the present perspective such corrections are not appropriate in the flat models and they are therefore omitted in the present paper.

In conclusion, the bounce phase itself does not lead to observed features of the CMB spectra. A post-bounce inflationary phase is required. Such a model with a bounce and the following inflationary phase already exists in loop quantum cosmology. In this case a non-vanishing scalar field potential is required and analytical solutions become unavailable. It is therefore the next step to investigate this model numerically.

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