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VORTEX IN AXION CONDENSATE AS A DARK MATTER HALO

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We study the possibility of the vortex formation in axion condensates on the galactic scale. Such vortices can occur as a result of global rotation of the early universe. We study analytical models of vortices and calculate exemplary galaxy rotation curves. Depending on the setup it is possible to obtain a variety of shapes which give a good qualitative agreement with observational results. However, as we show, the extremely low velocity dispersions of the axion velocities are required to form the single vortex on the galactic scales. We find that the required velocity dispersion is of the order of $\sigma \approx 10^{-12}$ m s^{-1}. This is much smaller that predicted within the present understanding of the axion physics. The vortices in axion condensate can however be formed on the much smaller scales and give seeds to the galaxy formation and to their angular momenta. On the other hand, the vortices can be formed on the galactic scales, but only if the mass of the axion-like particles is of the order of 10^{-30} eV. In this case, the particle de Broglie wavelength is comparable with the galactic diameter. This condition must be fulfilled in order to keep the coherence of the quantum condensate on galactic scales.

Keywords: Axion condensate; dark matter; cosmology.

1. Introduction

The problem of dark matter seems to be a challenge for theoretical and observational investigations. Recent astronomical observations, like the measurements of cosmic microwave background (CMB) radiation by WMAP,[1] or measurements of distant supernova Type Ia (SNIa),[2,3] indicate that the universe, apart from matter, is fulfilled with a phenomenological fluid with negative pressure (called dark energy) which could be responsible for the current acceleration of the universe. While the nature of this energy is still unknown (the cosmological constant Λ is the most serious candidate), the combination of results from CMB measurements, SNIa data and extragalactic observations indicate that as much as 2/3 of the total energy density of the universe is in the form of the mysterious dark energy. While it dominates the dynamics of the universe on the large scale, dark matter (matter whose existence has been inferred only through its gravity) clearly influences the dynamics on the galactic scale. Its abundance is given in terms of the density parameter $\Omega_{\rm DM} = \rho_{\rm DM}/\rho_{\rm c}$, where $\rho_{\rm c}$ is the critical energy density, $\rho_{\rm c} = 3H^2/8\pi G$. For the "concordance" flat ΛCDM model we have total matter $\Omega_{\rm m} \simeq 0.3$. In turn from emission and absorption of photons visible matter is roughly $\Omega_{\rm vis} \simeq 0.04$, giving us that dark matter amounts to $\Omega_{\rm DM} \simeq 0.26$.

The rotation curves of spiral galaxies give us the strongest evidence for dark matter. This dark matter in its 80% is in some form cold and nonbaryonic. Therefore, we can identify only 4% of the total matter content in the present universe. If we do not postulate the existence of dark matter it is not possible to explain why at a large distance r from the center of a given galaxy, we would find the circular velocity $v_c^2 \simeq GM_{\rm vis}/r$, since visible matter is concentrated around its center. For the observational results of rotation curves see Ref. 4. However, observations show that v_c is independent of r at large distances ($v_c \sim 200\,{\rm km\ s}^{-1}$ is its typical value). From numerical simulations of the halo formation we obtain density profiles for both small and large values $\rho_{\rm halo} \propto r^{-\alpha}$ with $\alpha \in (1; 1.5)$ for small and $\alpha = 3$ for large distances.[5,6] The flat part of the rotation curves corresponds to $\rho_{\rm halo} \propto r^{-2}$. Such a behavior can be explained e.g. within the isothermal sphere model of dark matter halo. In this model, the constant value of the rotation velocity v_c can be related with the velocity dispersion σ of the dark matter particles by $v_c = \sqrt{2}\sigma$.

There are two main candidates for cold dark matter, namely the axion and neutralino.[7–10] In this paper we concentrate on the axion. Axions were originally proposed to solve the strong CP problem in quantum chromodynamics (QCD). If axions have low mass, thus preventing other decay modes, axion theories predict that the universe would be filled with cold Bose–Einstein condensates of primordial axions.[11,12] The axions in this condensate are always nonrelativistic and if the mass is about 10^{-3} eV it would plausibly explain the dark matter problem. There are prospects for direct experimental detection of axions. The Axion Dark Matter eXperiment (ADMX)[13] searches for weakly interacting axions in the dark matter halo of our galaxy. Unfortunately, studies of axion dark matter are not sufficiently sensitive to probe the mass regions where axions would be expected.

We investigate the possibility of explaning flat velocity curves of spiral galaxies in terms of axion condensate which can be present in the universe since the Peccei–Quinn phase transition.

In our investigation we use the Gross–Pitaevski equation in an expanding FRW universe

$$i\hbar\left(\frac{\partial}{\partial t}+\frac{3}{2}\frac{\dot{a}(t)}{a(t)}\right)\phi(\mathbf{r},t)=\left(-\frac{\hbar^2}{2m}\frac{1}{a^2(t)}\nabla^2+U(\mathbf{r})+g^2|\phi(\mathbf{r},t)|^2\right)\phi(\mathbf{r},t), \qquad (1)$$

where $U(\mathbf{r})$ is the external potential, g^2 is the coupling constant between axions and $a(t)$ is the scale factor. Here we describe the condensate by the one-particle wavefunction $\phi(\mathbf{r},t)$. The circulation in condensate is expressed by[14]

$$\Gamma=\oint_C \mathbf{v}\cdot d\mathbf{r}=\frac{\hbar}{m}2\pi l, \qquad (2)$$

where l is an integer called the topological charge. C denotes any contour around a vortex. When there is no vortex, $l=0$ and the circulation vanishes. When the condensate is inside a rotating environment the vortex is formed. In the interacting condensate vortices that are with $l>1$ are unstable and decay to the vortices that are with $l=1$. So in a realistic situation we have a net of elementary vortices ($l=1$) which are stable. Such nets of vortices are observed in laboratories.[15]

We study the possibility that a galactic halo is just such a vortex. We suppose that such a mechanism can be realized in the early universe. Namely, in the presence of global rotation, the proposed mechanism yields a huge amount of small vortices whose topological charges are equal to 1. In this paper we consider a singular vortex in the axion condensate.

The question whether rotation is an attribute of the universe as a whole has been investigated since classical works of Lanczos,[16] Gamov,[17] Godel,[18] Hawking[19] and recently by Chapline and Mazur.[20] When compared with the CMB anisotropies, the effects of rotation should not be big today.[21] Barrow *et al.*[22] showed that the cosmic vorticity depends strongly on the cosmological model, and that for a flat universe the bound on vorticity relative to the Hubble parameter at present is $\omega/H=2\times10^{-5}$.

2. Free Axion Condensate

The Bose–Einstein occurs when the ground state becomes occupied by the macroscopic number of particles. Since each of these particles is in the same quantum state ϕ, the density of the condensate composed of the N particles of mass m can be expressed as $\rho=mN|\phi|^2$. Therefore, the square modulus of the wave function can be observed just as the density distribution of the particles in the condensate. The condition for condensation to occur is that the de Broglie wavelength of the particles must be comparable with the interparticles distances. The individual wavefunctions of the particles will overlap then. This requirement can be translated into the critical temperature T_c, below which the condensation starts. For $T<T_c$ phase–space density occupation number becomes significant. It is also important

to note that condensation does not occur for photons (and some other particles) because the number of photons is not conserved. Therefore during a cooling, the number of photons can be e.g., absorbed by the environment and condensation does not take place.

The axion field can be regarded as massless at the temperatures $T \geq 1$ GeV. In turn, at the lower temperatures, $T \leq 1$ GeV, the axion field becomes massive due to the nonperturbative QCD effects. The oscillations of this field are interpreted as the massive axion particles. The axion mass can be expressed as follows[23]

$$m_a \simeq 6\mu\text{eV} \left(\frac{10^{12}\,\text{GeV}}{f_a} \right), \tag{3}$$

where f_a is the energy of $U_{PQ}(1)$ symmetry breaking and creation of axions. Here f_a is a free parameter; however, its value is empirically constrained to 10^9 GeV $\lesssim f_a \lesssim 10^{12}$ GeV, corresponding to the axion mass window $6 \cdot 10^{-6}$ eV $\lesssim m_a \lesssim 6 \cdot 10^{-3}$ eV. In the subsequent part of the paper we restrict to the case $f_a = 10^{12}$ GeV what leads to $m_a \approx 10^{-5}$ eV.

Because axions that are massless at the energies $T \geq 1$ GeV, the condensation cannot take place until the universe cools down to $T \sim 1$ GeV. As it was shown in Ref. 11, the axion temperature is then much smaller than T_c what allows for the Bose–Einstein condensation to occur. Namely the temperature is of the order of the Hubble expansion rate $T \sim 10^{-3}$ eV while $T_c \sim 300$ GeV. Since the axions are very weakly coupled, they can be considered as a gas of free particles. Therefore in the first approximation the axion condensate can be described by the Gross–Pitaevski equation (1) with $g = 0$ and $U(\mathbf{r}) = 0$. However, in order to trace the thermalization of the Bose–Einstein condensate, the weak gravitational coupling between axions must be also taken into account. In this section, we neglect this coupling and consider the free axion condensate as formed at the QCD epoch for $T \sim 1$ GeV. Namely, when the axion acquired a mass and the conditions for the Bose–Einstein condensation were satisfied.

In this case, the free Gross–Pitaevski equation (1) takes the form

$$i\hbar \left(\frac{\partial}{\partial t} + \frac{3}{2}\frac{\dot{a}(t)}{a(t)} \right) \phi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\frac{1}{a^2(t)} \left[\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} - \frac{l(l+1)}{r^2} \right] \phi(\mathbf{r},t). \tag{4}$$

The general solution of this equation is in the form

$$\phi(\mathbf{r},t) = \frac{C}{a^{3/2}} \exp\left\{ -\frac{i}{\hbar} \int \frac{\mu}{a^2(t)} dt \right\} j_l(\sqrt{\lambda} r)\, Y_l^n(\theta,\varphi), \tag{5}$$

where C is the normalization constant and $\lambda = 2m\mu/\hbar^2$. The energy of the particle is defined as $E = \mu/a^2$. Here $j_l(x)$ is the spherical Bessel function related to the ordinary one with

$$j_l(x) = \sqrt{\frac{\pi}{2x}}\, J_{l+\frac{1}{2}}(x), \tag{6}$$

and Y_l^n are the standard spherical harmonics. Based on (5), the energy density of the axion condensate, composed of N axions, can be written as follows:

$$\rho(\mathbf{r},t) = mN|\phi(\mathbf{r},t)|^2 = \frac{mN|C|^2}{a^3}\, j_l^2(\sqrt{\lambda}r)|Y_l^n(\theta,\varphi)|^2, \tag{7}$$

where m is axion mass. As we see, during the cosmological evolution, the axion condensate is diluted just as ordinary dust matter, namely $\rho \sim 1/a^3$.

The solution (5) is non-normalizable, therefore the constant C cannot be directly evaluated. To obtain a normalizable solution we must introduce proper boundary conditions, for example $\phi(r \geq \bar{r}) = 0$ for some $\bar{r}$. But it is not natural in our case, as our universe is filled homogeneously by the axion condensate. Free axions should thus be described approximately by a non-normalizable wave with defined energies. Accordingly, our solution describes an axion condensate with defined non-quantized energy.

Our main idea is that small vortices created in the early universe can grow during expansion of the universe. At the early stage, the vortex is a quantum object, but during expansion it can becomes a classical object. The expansion of the free condensate follows directly the expansion of the universe. This might not be a case for the self-interacting condensate. Then, the interaction resists the expansion due to the cosmic expansion. When the quantum system is strongly bounded, it can be even insensitive to the cosmic expansion. It is similar to the case for the hydrogen atom which energy levels remain unaffected by the cosmic expansion. For the free condensate, the spatial part (in coordinate variables) decouples from the temporal dependence. Therefore if the wavefunction has some characteristic scale, say r_x, the the corresponding physical scale will accordingly be $R_\text{x} = ar_\text{x}$. Therefore whole physical scales of the wavefunction will expand homogeneously. In order to find the value of this growth we have to determine the total expansion of the universe from the formation of axion condensates untill now ($a(t_0) = 1$). Axion condensates were formed at some $a(t_1)$ what we take to be $T = 1$ GeV. The corresponding value of time is $t_1 \simeq 2 \cdot 10^{-7}$ s. Based on this, we have

$$\frac{a(t_0)}{a(t_1)} = \frac{1\text{GeV}}{T_\text{rec}}(1 + z_\text{rec}) \simeq 5 \cdot 10^{12}, \tag{8}$$

where the $T_\text{rec} = 0.2$ eV is the energy scale of recombination, and $z_\text{rec} \simeq 10^3$ is the corresponding value of redshift. So if the present characteristics size of the axion wavefunction is of order of the galactic scale $R_\text{x} = 10$ kpc, it corresponds to $R_\text{x} \simeq 10^8$ m at t_1.

Let us firstly examine the case without the vortex, $l = 0$. The solution of Gross–Pitaevski equation is of the form

$$\phi(\mathbf{r},t) = \frac{C}{\sqrt{4\pi}}\frac{1}{a^{3/2}}\exp\left\{-\frac{i}{\hbar}\int \frac{\mu}{a^2(t)}dt\right\}\frac{\sin(\sqrt{\lambda}r)}{\sqrt{\lambda}r}. \tag{9}$$

This solution describes a spherically symmetrical halo with the density distribution

$$\rho(r,t) = mN|\phi(\mathbf{r},t)|^2 = \frac{mN|C|^2}{4\pi a^3}\left[\frac{\sin\left(\sqrt{\lambda}r\right)}{\sqrt{\lambda}r}\right]^2. \tag{10}$$

The density in the central region is given by

$$\rho(r\to 0,t) = \frac{mN|C|^2}{4\pi a^3} := \rho_0. \tag{11}$$

We must remember that the coordinate r in not a physical distance, which is $R = a\cdot r$. For a spherical distribution we calculate the velocity rotation curve from the relation

$$v(R) = \sqrt{\frac{G\mathcal{M}(R)}{R}}, \tag{12}$$

where $\mathcal{M}(R)$ is the mass function and is expressed as

$$\mathcal{M}(R) = 4\pi\int_0^R R'^2\,\rho(R')\,dR'. \tag{13}$$

Based on this, we find

$$v(R) = v_0\sqrt{1-\frac{\sin(R/R_0)}{R/R_0}} \tag{14}$$

where

$$v_0 = \sqrt{2\pi G\rho_0\frac{a^2}{\lambda}} = \hbar\sqrt{\frac{\pi G\rho_0}{mE}}, \tag{15}$$

$$R_0 = \frac{a}{2\sqrt{\lambda}} = \frac{\hbar}{2\sqrt{2mE}}. \tag{16}$$

Figure 1 shows the velocity curve for axion condensate (14). We compare the theoretically predicted curves with the exemplary galactic rotation curves.[24]

This result describes the observed galaxy velocity curves quite well — we can see the characteristic change to a plateau in the rotation curves. This is not seen in the comparison with NGC3621 galaxy. However even in this case, the main observational features can be reproduced.

Kinetic energy of the axion particles is equal to $E = m\sigma^2/2$, where σ is the velocity dispersion. Based on this, the Eqs. (15) and (16) can be rewritten to the following form

$$v_0 = \frac{\hbar}{m\sigma}\sqrt{2\pi G\rho_0}, \tag{17}$$

$$R_0 = \frac{1}{2}\frac{\hbar}{m\sigma}. \tag{18}$$

Therefore, based on the observationally determined values of R_0 and v_0, the quantities ρ_0 and the product $m\sigma$ can be determined. However, the values of m and

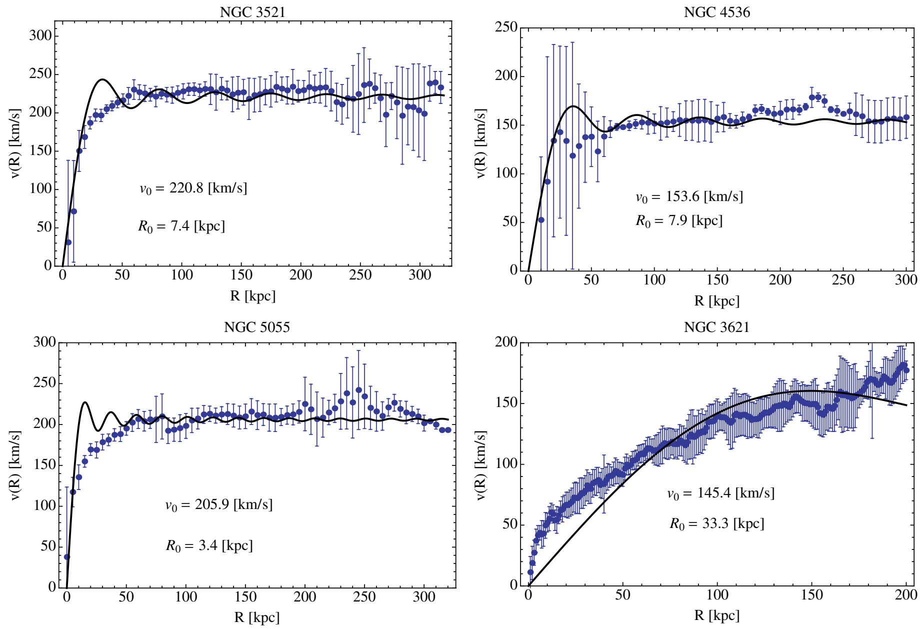

Fig. 1. The contribution to the galactic velocity curves given by the axion condensate. The data points are galactic velocity curves of the spiral galaxies. The NGC3521, NGC4536, NGC3621 are barred spiral galaxies (SBbc). The NGC5055 is the spiral galaxy with the central bulge (SbSc).

σ cannot be determined independently. Combining the above equations, we find the consistency relation for our model

$$\rho_0 = \frac{1}{8\pi G}\frac{v_0^2}{R_0}. \tag{19}$$

Therefore, the energy density in the central region of the halo can be determined based on R_0 and v_0 determined form the observations. This quantity can be compared with the observational value that is determined independently.

The value of velocity dispersion of axions can be expressed in the following way:

$$\sigma \approx \frac{\hbar}{mct_1}\frac{a(t_1)}{a(t)}. \tag{20}$$

This expressions comes from the assumption that the axion momenta at the time t_1 is comparable with the corresponding Hubble factor. Therefore $p = m\sigma \approx H = 1/2t_1$, the factor $a(t_1)/a(t)$ is due to the cosmological redshift. The expression does not take into account the contribution due to gravitationally induced thermalization. The interactions between the axions are neglected here. Based on this, the expression on the parameter R_0 takes the form

$$R_0 \approx \frac{ct_1}{2}\frac{a(t)}{a(t_1)}. \tag{21}$$

Based on this, the present value of R_0 is predicted to be $R_0 \approx 10^{-2}$ pc. This value is six orders of magnitude lower than the typically observed value $R_0 \approx 10$ kpc. Therefore, in order to explain the galactic rotation curves by the axion condensate, the velocity dispersion σ (or the product σm if the mass is not fixed) must be smaller than what is predicted from (20). Namely for $R_0 \approx 10$ kpc, based on (18) with $m_a \approx 10^{-5}$ eV, we find $\sigma \approx 10^{-12}\ m/s$. Therefore, the dispersion of the axions velocities must be extremely low in order to explain the galactic halo in terms of the wavefunction of the axion condensate. This velocity is much smaller than the prediction from Eq. (20). Moreover, the additional velocity dispersion can be produced due to the gravitational interactions. Therefore we conclude that, the axions of the mass order of $m_a \approx 10^{-5}$ eV cannot form the quantum compact structures on the galactic scales. They can however form the structures on the smaller scales.

Based on (18) one can answer, what the necessary mass of the axion-like particle should be to produce the quantum condensate on galactic scales. In order to answer this we have to assume some velocity dispersion of the dark matter particles. If they thermalize due to the gravitational interactions, the velocity dispersion should be of the order of the radial velocity of the stars (as in the isothermal sphere model). Taking $\sigma \approx 10^2$ km/s and $R_0 \approx 10$ kpc we find

$$mc^2 = \frac{\hbar c}{2R_0}\frac{c}{\sigma} \approx 10^{-30}\ \text{eV}. \tag{22}$$

The particle must me therefore ultralight. The interpretation of this result is clear when we recognize the right side in Eq. (18). It is just, up to a numerical factor, the de Broglie wavelength λ_{dB} of the axion-like particle. These wavelengths have to be of the galactic sizes in order to keep the coherence of the quantum system. Otherwise the whole structure will disintegrate due to decoherence. Here this condition naturally emerges. Namely, Eq. (18) can be rewritten into the form

$$\lambda_{\text{dB}} = 4\pi R_0, \tag{23}$$

where the right side is of the order of galactic diameter.

3. Vortex in the Free Axion Condensate

Similar results can be obtained by considering the vortex solution with $l = 1$. The velocity curves also exhibit a plateau, like the solution with $l = 0$. The only difference is that we now have the angle dependence $\rho \sim \cos^2\theta$. This produces, like for each vertex solution, distortions from the spherical shape of a dark matter halo.

The $l = 1$ is a stable vortex configuration. Taking

$$Y_1^1(\theta,\varphi) = \frac{1}{\sqrt{2\pi}}\frac{\sqrt{6}}{2}\cos\theta e^{i\varphi}, \tag{24}$$

we find the corresponding solution

$$\phi(\mathbf{r},t) = \frac{C\sqrt{3}}{2\sqrt{\pi}}\frac{1}{a^{3/2}}\exp\left\{-\frac{i}{\hbar}\int\frac{\mu}{a^2(t)}dt\right\}\left(\frac{\sin(\sqrt{\lambda}r)}{(\sqrt{\lambda}r)^2} - \frac{\cos(\sqrt{\lambda}r)}{\sqrt{\lambda}r}\right)\cos(\theta)e^{i\varphi}. \tag{25}$$

In similarity with the $l = 0$ case we find the density distribution

$$\rho(r,\theta,t) = 3\rho_0 \underbrace{\left[\frac{\sin(\sqrt{\lambda}r)}{(\sqrt{\lambda}r)^2} - \frac{\cos(\sqrt{\lambda}r)}{\sqrt{\lambda}r}\right]^2}_{=j_1^2(\sqrt{\lambda}r)} \cos^2\theta. \tag{26}$$

We have used here the definition of ρ_0 introduced in the $l = 0$ case. However, here one cannot interpret it as a density for $r \to 0$, because now $\rho(r \to 0) = 0$. Note, that in the case considered, the density distribution strongly depends on the angle θ, namely $\rho \propto \cos^2\theta$. Therefore, at the surface of galactic disc ($\theta = \pi/2$) the density of dark matter halo vanishes. This dependence must be taken into account while calculating the corresponding velocity curves. In general, one should firstly calculate the surface density Σ, and integration density $\rho(r,\theta,t)$ over the z direction. Defining $\mathcal{R}$ as a physical radial variable on the surface $z = 0$ we find

$$\Sigma(\mathcal{R}) = 3\rho_0 \int_{-\infty}^{+\infty} dz j_1^2\left(\frac{\sqrt{\lambda}}{a}\sqrt{z^2+\mathcal{R}^2}\right)\frac{z^2}{z^2+\mathcal{R}^2}. \tag{27}$$

We were however unable to compute this integral analytically. Based on this one can derive the mass function

$$\mathcal{M}(\mathcal{R}) = 2\pi \int_0^{\mathcal{R}} \mathcal{R}'\Sigma(\mathcal{R}')d\mathcal{R}', \tag{28}$$

and then the corresponding velocity curve

$$v(\mathcal{R}) = \sqrt{\frac{G\mathcal{M}(\mathcal{R})}{\mathcal{R}}}. \tag{29}$$

Since this method is computationally rather impractical, we can alternatively integrate over the angular part, then

$$\rho(r,t) = \int \rho(r,\theta,t)\, d\Omega = 4\pi\rho_0 \left[\frac{\sin(\sqrt{\lambda}r)}{(\sqrt{\lambda}r)^2} - \frac{\cos(\sqrt{\lambda}r)}{\sqrt{\lambda}r}\right]^2. \tag{30}$$

With the use of the expression on R (without the factor 4π, since we have already integrated over the solid angle) and (12) we find

$$v(R) = v_0 \frac{R_0}{R}\sqrt{4\left[\cos(R/R_0) - 1\right] + (R/R_0)^2 + (R/R_0)\sin(R/R_0)}, \tag{31}$$

where v_0 and R_0 are defined similarly to the $l = 0$ case. In Fig. 2 we compare expression (31) with the analogous function previously found in the $l = 0$ case. As we see there is no qualitative difference between the velocity curves for $l = 1$ and $l = 0$. In both cases, the velocity curves become flat, $v(R) \approx v_0$, for $R \gg R_0$. In Fig. 3 we compare the velocity curve (31) with the exemplary galactic velocity curve. The fit gives a qualitatively good explanation of the observational data. Despite of this, as we have discussed before, the quantum structure on the galactic scales can be formed only if the mass of axion-like particle is extremely low. These results naturally apply for the case of single vertex considered in this section. The vortex

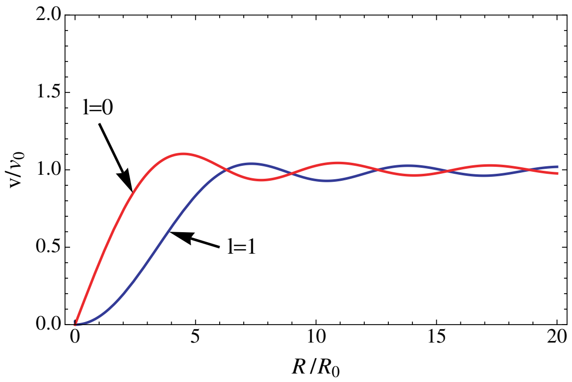

Fig. 2. A comparison of the radial velocity curves for $l = 1$ and $l = 0$. In both cases, the velocity curves are flat, $v(R) \approx v_0$, for $R \gg R_0$.

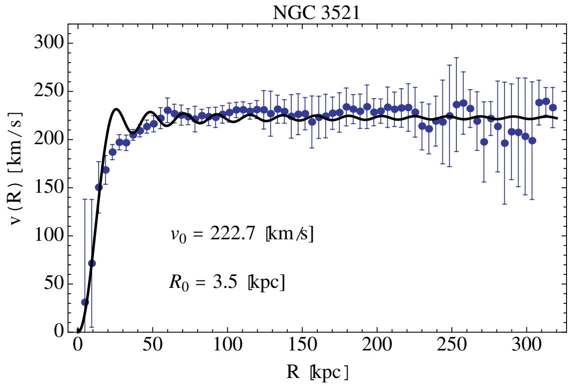

Fig. 3. Radial velocity curve (31) for the axion condensate with $l = 1$ compared with the observational data for the galaxy NGC 3521. In general, we observe that there is a good agreement between the theoretical curve and the observational data. The main difference is the first peak of the theoretical curve. The predicted radial velocity is at this point about 30 km/s higher than the observed value.

in axion condensates cannot form on the galactic scales because the axion mass is to high. However, the vortices can be formed on the much smaller scales. As we have previously estimated, $R \sim 10^{-2}$ pc, neglecting the gravitational interactions of the axion particles. Such small vortices can seed galaxy formation. They can also be a source of the angular momenta of the primordial galaxies. This possibility is an attractive subject for the further studies. Therefore the concept of axion vortices can be still potentially useful in explaining formation of galaxies.

4. Discussion

We have considered axion condensates as a candidate for dark matter in galactic halos. Condensates are known to exhibit vortices induced by the rotation of the environment in laboratories. Such vortices are unstable and decay into states with the lowest angular moment. We prolong this picture onto the entire universe whose global rotation might give rise to the rotation of particular galaxies. Precisely, the total angular momenta can decay onto the elementary vortices, each with the same topological charge $l = 1$.

We have considered the non-normalizable states of the axion condensates with $l = 0$ and $l = 1$ (vortex). It was shown that these solutions lead to the flat velocity curves characterized by oscillations in a plateau region. There was no qualitative difference between the velocity curves obtained in these two cases. We have confronted the theoretical curves with the exemplary observational data. We found that the theoretical curves reproduce the observational data. However, we have to keep in mind that we neglected contribution from the ordinary luminous matter. Therefore the velocity curves obtained give only the contribution form the dark matter halo. The contribution from the luminous matter can be important near the galactic center, but is secondary at the larger distances. Therefore, the flatness of the velocity curves at $R \gg R_0$ will be conserved even in presence of this additional matter content.

In this paper we have restricted to the case of the free axion condensate. Some tiny interaction can be present due to the gravitational forces. These interaction can be in fact important from the perspective of thermalization of the axion condensate. It requires further numerical studies to take into account the gravitational interactions in the considered scenario. In particular, the axion condensate will not expand freely during cosmological evolutions, as it was observed in case of the free axion condensate.

Another issue is that the axions have some velocity dispersion. Because of this, the individual wavefunctions of the axions differ a bit, and the wavefunction of the condensate should be considered as a superposition

$$\Psi(\mathbf{x}, t) = \int d^3\mathbf{k} f(k)\phi_k(\mathbf{x}, t), \tag{32}$$

where $\phi_k(\mathbf{x}, t)$ are eigenfunctions of the Hamiltonian. The function $f(k)$ is the bell-shaped function centered at $k = 0$ and with the dispersion related to σ. This superposition of non-normalizable states will lead to the normalizable state $\Psi(\mathbf{x}, t)$.

The considered model has however serious limitations if applied to axions. Namely, the velocity dispersion of the axions must be much smaller than predictions from the theory. Moreover, the gravitational interaction of the axion will lead to velocity dispersions much higher than required to form the quantum vortex on galactic scales. Intuitively, the de Broglie wavelength of axions is much smaller than the galactic scales. Therefore the quantum condensate will not keep the coherence on such scales. However, it does not exclude the axions as dark matter particles forming the galactic halo. Due to the gravitational interactions they thermalize and can be described by the model of isothermal sphere. In this case, the flat velocity curves are also explained, and the constant velocity of galaxies is related to the velocity dispersion of axions by the relation $v_c = \sqrt{2}\sigma$. The vortices in axion condensate can be however important in the process of the galaxy formation. Namely they can give the initial conditions for the galaxy formation. In particular, they can be responsible for the initial angular moment. This may also explain why the masses of the spiral galaxies are similar and equal to $\sim 10^{11}$ $M_\odot$. This can be possibly explained since all the vortices have the same topological charge $l = 1$, which is directly a quantum effect. Therefore, initially, galaxies can grow on the independent vortices in the axion condensate. But later, when the accretion of matter (including other axions) proceed, the length of coherence of the individuals vortices became much smaller than the length of created gravitating structure. The vortices in the axion condensate can therefore be seeds of the galaxies but have no important influence on them at the further stages. However, the idea that the vortices in the axion condensate could have something to do with the galaxy formation must be approved in further investigations.

Alternatively, the coherent quantum structure such as the vortices, can be formed on the galactic distances, but only if the masses of the axion-like particles are extremely low, namely of the order of 10^{-30} eV. In such a case the de Broglie wavelength of these particles is comparable with the galactic sizes. This condition prevents the disintegration of the quantum system due to the lack of coherence between the different parts of the quantum galactic halo. Concluding, the explanation of the galactic velocity curves presented in this paper can still hold, but only if the dark matter particles are ultralight. The axions of this mass are however, illicit.

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References

1. E. Komatsu *et al.*, [arXiv:1001.4538 astro-ph.CO].
2. A. Riess *et al.*, *Astron. J.* **116** (1998) 1009.
3. S. Perlmutter *et al.*, *Astrophys. J.* **517** (1999) 565.
4. M. Persic, P. Salucci and F. Stel, *Mon. Not. Roy. Astron. Soc.* **281** (1996) 27.
5. B. Moore *et al.*, *Mon. Not. Roy. Astron. Soc.* **310** (1999) 1147.
6. J. F. Navarro, C. S. Frenk and S. D. M. White, *Astrophys. J.* **490** (1997) 493.
7. G. Lazarides, [arXiv:hep-ph/0601016].
8. P. Gondolo, *NATO Sci. Ser. II* **187** (2005) 279.
9. L. Visinelli and P. Gondolo, *Phys. Rev. D* **80** (2009) 035024.
10. J. C. Hwang and H. Noh, *Phys. Lett. B* **680** (2009) 1.
11. P. Sikivie and Q. Yang, *Phys. Rev. Lett.* **103** (2009) 111301.
12. E. W. Mielke and J. A. V. Perez, *Phys. Lett. B* **671** (2009) 174.
13. L. D. Duffy *et al.*, *Phys. Rev. D* **74** (2006) 012006.
14. S. Ghosh, *Phase Transit.* **77** (2004) 625.
15. F. Chevy *et al.*, [arXiv:cond-mat/0104218].
16. K. Lanczos, *Z. Phys.* **21** (1924) 73.

17. G. Gamov, *Nature* **158** (1946) 549.
18. K. Godel, *Rev. Mod. Phys.* **21** (1949) 447.
19. S. W. Hawking, *Mon. Not. Roy. Astron. Soc.* **142** (1969) 129.
20. G. Chapline and P. O. Mazur, *AIP Conf. Proc.* **822** (2006) 160.
21. J. Silk, *Mon. Not. Roy. Astron. Soc.* **147** (1970) 13.
22. J. D. Barrow, R. Juszkiewicz and D. Sonoda, *Mon. Not. Roy. Astron. Soc.* **213** (1985) 917.
23. P. Sikivie, [arXiv:astro-ph/0610440].
24. http://www.astro.umontreal.ca/fantomm/sings/rotation_curves.htm