Niegeometryczny opis grawitacji

część II

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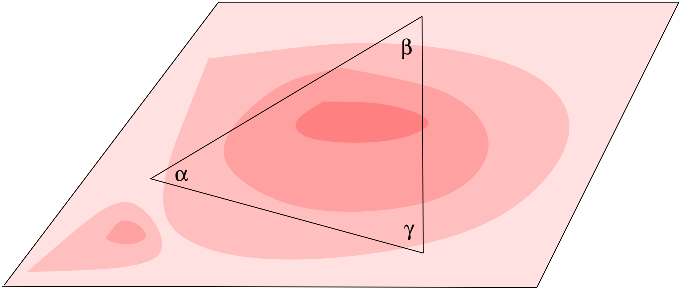


$$\alpha + \beta + \gamma = 180^\circ$$

- Propagator dla pola swobodnego
- Dlaczego spada ?
- Efekt Tolmana-Ehrenfesta-Podolsky'ego
- Stała kosmologiczna
- Rozbieżności

Propagator dla pola swobodnego

Elektrodynamika:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\zeta(\partial_\sigma A^\sigma)^2$$

Dla $\zeta = 1$ (cechowanie Feynmana)

$$\mathcal{L} \to -\partial_\mu A_\nu \partial^\mu A^\nu \text{ ''}=\text{'' } +\frac{1}{2}A_\mu \Box A^\mu$$

Grawitacja zlinearyzowana:

$$\mathcal{L} = \frac{1}{2}h_{\mu\nu,\sigma}h^{\mu\nu,\sigma} - h^{\mu\nu}{}_{,\nu}h^{\sigma}{}_{\mu,\sigma} + h^{\mu\nu}{}_{,\nu}h^{\sigma}{}_{\sigma,\mu} - \frac{1}{2}h^{\nu}{}_{\nu,\mu}h_{\sigma}{}^{\sigma,\mu} - \lambda T^{\mu\nu}h_{\mu\nu}$$

Ustalamy cechowanie

$$\mathcal{L} \to \mathcal{L} + (C_\nu)^2$$

gdzie

$$C_\nu = \partial^\mu h_\mu - \frac{1}{2}\partial_\nu h = \partial^\mu \underbrace{\left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h\right)}_{\bar{h}_{\mu\nu}} = \partial^\mu \bar{h}_{\mu\nu}$$

dostajemy

$$\mathcal{L} = \frac{1}{2}\partial_\lambda h_{\mu\nu}\partial_\lambda h^{\mu\nu} - \frac{1}{4}\partial_\lambda h \partial^\lambda h - \lambda T^{\mu\nu} h_{\mu\nu}$$

lub

$$\mathcal{L} = \partial_\lambda h_{\alpha\beta} V^{\alpha\beta\mu\nu} \partial_\lambda h_{\mu\nu} - \lambda T^{\mu\nu} h_{\mu\nu}$$

gdzie

$$V^{\alpha\beta\mu\nu} = \frac{1}{2}\eta^{\alpha\mu}\eta^{\beta\nu} - \frac{1}{4}\eta^{\alpha\beta}\eta^{\mu\nu}$$

$$\mathcal{L} = \partial_\lambda h_{\alpha\beta} V^{\alpha\beta\mu\nu} \partial_\lambda h_{\mu\nu} - \lambda T^{\mu\nu} h_{\mu\nu}$$

Zastępujemy pole $h_{\mu\nu}$ polem Ψ^i według przepisu

Zamiana $h \to \Psi$:

h	11	22	33	44	12	13	14	23	24	34
Ψ	1	2	3	4	5	6	7	8	9	10

Dostajemy

$$\mathcal{L} = \partial_\mu \Psi^i V^{ij} \partial^\mu \Psi^j - \lambda \Psi^i T^i$$

A stad równania ruchu

$$2V^{ij} \Box \Psi^i = -\lambda T^j$$

$$\mathcal{L} = \partial_\mu \Psi^i V^{ij} \partial^\mu \Psi^j - \lambda \Psi^i T^i$$

$$V^{ij} = \begin{pmatrix} +\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{4} & +\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{4} & -\frac{1}{4} & +\frac{1}{4} & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & +\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$2V^{ij}\Box\Psi^i = -\lambda T^j$$

Czyli równanie na funkcję Greena ma postać

$$V^{ij}\Box_x D^{jk}(x-y) = -\frac{1}{2}\delta^{ik}\delta(x-y)$$

Podstawiając

$$\begin{aligned} D^{jk}(x-y) &= \int \frac{d^4k}{(2\pi)^4} D^{jk}(k) e^{ik(x-y)} \\ \delta(x-y) &= \int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)} \end{aligned}$$

Dostajemy

$$\begin{aligned} -k^2 \mathbf{V}\mathbf{D}(k) &= -\frac{1}{2}\mathbf{1} \\ \mathbf{D}(k) &= \frac{1}{2}\frac{1}{k^2}\mathbf{V}^{-1} \end{aligned}$$

$$(V^{-1})^{ij} = \begin{pmatrix} +1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & +1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & +1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & +1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Przyporządkowując

$$\begin{aligned} i &\rightarrow \mu\nu \\ j &\rightarrow \lambda\sigma \end{aligned}$$

dostajemy

$$(V^{-1})_{\mu\nu;\lambda\sigma} = \eta_{\mu\lambda}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\lambda} - \eta_{\mu\nu}\eta_{\lambda\sigma}$$

i wstawiając do

$$D_{\mu\nu;\lambda\sigma}(k) = \frac{1}{2}\frac{1}{k^2}(V^{-1})_{\mu\nu;\lambda\sigma}$$

otrzymujemy

Propagator grawitonu:

$$D_{\mu\nu;\lambda\sigma}(k) = \frac{1}{2}\frac{\eta_{\mu\lambda}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\lambda} - \eta_{\mu\nu}\eta_{\lambda\sigma}}{k^2 + i\epsilon}$$

Dlaczego spada ?

$$Z \equiv \langle 0|e^{-iHT}|0\rangle = \int \mathcal{D}h_{\mu\nu}e^{iS[h_{\mu\nu}]}$$

gdzie

$$S[h_{\mu\nu}] = \int d^4x \left(\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial_\lambda h^{\mu\nu} - \frac{1}{4}\partial_\lambda h \partial^\lambda h - \lambda T^{\mu\nu}h_{\mu\nu}\right)$$

wstawiamy

$$\begin{aligned} \frac{1}{2}\partial_\lambda h_{\mu\nu}\partial_\lambda h^{\mu\nu} &= \frac{1}{2}\partial_\lambda(h_{\mu\nu}\partial_\lambda h^{\mu\nu}) - \frac{1}{2}h_{\mu\nu}\Box h^{\mu\nu} \\ -\frac{1}{4}\partial_\lambda h\partial^\lambda h &= -\frac{1}{4}\partial_\lambda(h\partial^\lambda h) + \frac{1}{4}h\Box h \end{aligned}$$

Dostajemy

$$Z = \int \mathcal{D}h_{\mu\nu} e^{i\int d^4x\left(-\frac{1}{2}h_{\mu\nu}\Box h^{\mu\nu}+\frac{1}{4}h\Box h-\lambda T^{\mu\nu}h_{\mu\nu}\right)}$$

lub definiując

$$\begin{aligned} A_{\mu\nu;\lambda\sigma}(x,y) &= -\frac{1}{2}\delta(x-y)(\eta_{\mu\lambda}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\lambda}-\eta_{\mu\nu}\eta_{\lambda\sigma})\Box_y \\ J^{\mu\nu} &= -\lambda T^{\mu\nu} \end{aligned}$$

dostajemy

$$Z = \int \mathcal{D}h_{\mu\nu} e^{\frac{i}{2}\int d^4x d^4y h^{\mu\nu}(x)A_{\mu\nu;\lambda\sigma}(x,y)h^{\lambda\sigma}(y)+i\int d^4x J^{\mu\nu}(x)h_{\mu\nu}(x)}$$

W przypadku dyskretnym

$$
\begin{aligned}
Z &= \int_{-\infty}^{+\infty} d\phi_1 \dots d\phi_N e^{\frac{i}{2}\phi_i A_{ij}\phi_j + iJ_i\phi_j} \\
&= \sqrt{\frac{(2\pi i)^N}{\det A}} e^{-\frac{i}{2}J_i(A^{-1})_{ij}J_j} = \sqrt{(2\pi i)^N} e^{-\frac{1}{2}\mathrm{tr}\ln A - \frac{i}{2}J_i(A^{-1})_{ij}J_j}
\end{aligned}
$$

Przechodząc do granicy

$$
Z = Z[J=0]e^{-\frac{i}{2}\int d^4x d^4y J^{\mu\nu}(x) A^{-1}_{\mu\nu;\lambda\sigma}(x,y) J^{\lambda\sigma}(y)}
$$

Czym jest A^{-1} ?

$$
\int d^4 A(x,y)^{\mu\nu;\lambda\sigma} A^{-1}(y,z)_{\mu\nu;\lambda\sigma} = \delta(x-z)
$$

$$\int d^4 A(x,y)^{\mu\nu;\lambda\sigma} A^{-1}(y,z)_{\mu\nu;\lambda\sigma} = \delta(x-z)$$

wstawiając

$$A^{\mu\nu;\lambda\sigma}(x,y) = -\frac{1}{2}\delta(x-y)(\eta^{\mu\lambda}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\lambda} - \eta^{\mu\nu}\eta^{\lambda\sigma})\Box_y$$

dostajemy

$$\frac{1}{2}(\eta^{\mu\lambda}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\lambda} - \eta^{\mu\nu}\eta^{\lambda\sigma})\Box_x A^{-1}(x,z)_{\mu\nu;\lambda\sigma} = -\delta(x-z)$$

czyli

$$A^{-1}(x,z)_{\mu\nu;\lambda\sigma} = D_{\mu\nu;\lambda\sigma}(x-y)$$

$$Z = Z[J=0]e^{iW(J^{\mu\nu})}$$

gdzie

$$W(J^{\mu\nu}) = -\frac{1}{2}\int d^4x d^4y J^{\mu\nu}(x) D_{\mu\nu;\lambda\sigma}(x-y) J^{\lambda\sigma}(y)$$

$$Z \equiv \langle 0|e^{-iHT}|0\rangle = e^{-iET} = Z[J=0]e^{iW(J^{\mu\nu})}$$

Zbadajmy oddziaływanie pomiędzy dwoma punktowymi, spoczywającymi masami
Dla takiego układu tensor energii-pędu ma postać

$$T^{00}(\vec{x}) = m_1\delta(\vec{x}-\vec{x}_1) + m_2\delta(\vec{x}-\vec{x}_2)$$

pamiętamy że $J^{\mu\nu} = -\lambda T^{\mu\nu}$

Wstawiając

$$J^{00}(\vec{x}) = -\lambda[m_1\delta(\vec{x}-\vec{x}_1) + m_2\delta(\vec{x}-\vec{x}_2)]$$

oraz

$$D_{\mu\nu;\lambda\sigma}(x-y) = \int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)} \frac{1}{2} \frac{\eta_{\mu\lambda}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\lambda} - \eta_{\mu\nu}\eta_{\lambda\sigma}}{k^2 + i\epsilon}$$

do

$$W(J^{\mu\nu}) = -\frac{1}{2}\int d^4x d^4y J^{\mu\nu}(x) D_{\mu\nu;\lambda\sigma}(x-y) J^{\lambda\sigma}(y)$$

dostajemy

$$W(J^{\mu\nu}) = -\frac{1}{4}\lambda^2 \int d^4x d^4y \frac{d^4k}{(2\pi)^4} e^{ik(x-y)} \frac{1+1-1}{k^2+i\epsilon} \left[m_1^2\delta(\vec{x}-\vec{x}_1)\delta(\vec{y}-\vec{x}_1) + \right.$$
$$\left. + m_1 m_2\delta(\vec{x}-\vec{x}_1)\delta(\vec{y}-\vec{x}_2) + m_1 m_2\delta(\vec{y}-\vec{x}_1)\delta(\vec{x}-\vec{x}_2) + m_2^2\delta(\vec{x}-\vec{x}_2)\delta(\vec{y}-\vec{x}_2)\right]$$

$$W_{int} = -\frac{1}{4}\lambda^2 m_1 m_2 \int d^4x d^4y \frac{d^4k}{(2\pi)^4} \frac{e^{ik(x-y)}}{k^2} [\delta(\vec{x}-\vec{x}_1)\delta(\vec{y}-\vec{x}_2)+\delta(\vec{y}-\vec{x}_1)\delta(\vec{x}-\vec{x}_2)]$$

$$W_{int} = -\frac{1}{2}\lambda^2 m_1 m_2 \int dx^0 dy^0 \frac{dk^0}{2\pi} \frac{d^3k}{(2\pi)^3} \frac{e^{-i\vec{k}(\vec{x}_1-\vec{x}_2)}}{k_0^2-\vec{k}^2+i\epsilon} e^{ik^0(x^0-y^0)}$$

$$W_{int} = -\frac{1}{2}\lambda^2 m_1 m_2 \int dx^0 \delta(k^0) dk^0 \frac{d^3k}{(2\pi)^3} \frac{e^{-i\vec{k}(\vec{x}_1-\vec{x}_2)}}{k_0^2-\vec{k}^2+i\epsilon} e^{ik^0x^0}$$

$$W_{int} = \frac{1}{2}\lambda^2 m_1 m_2 \underbrace{\left(\int dx^0\right)}_{T} \int \frac{d^3k}{(2\pi)^3} \frac{e^{-i\vec{k}(\vec{x}_1-\vec{x}_2)}}{\vec{k}^2+i\epsilon}$$

$$-iE_{int}T = iW_{int}$$

$$E_{int} = -\frac{1}{2}\lambda^2 m_1 m_2 \int \frac{d^3k}{(2\pi)^3} \frac{e^{-i\vec{k}(\vec{x}_1 - \vec{x}_2)}}{\vec{k}^2 + i\epsilon}$$

$$\int \frac{d^3k}{(2\pi)^3} \frac{e^{-i\vec{k}(\vec{x}_1 - \vec{x}_2)}}{\vec{k}^2 + i\epsilon} = \frac{1}{4\pi r}$$

gdzie $r = |\vec{x}_1 - \vec{x}_2|$.

$$E_{int} = -\frac{1}{2}\lambda^2 m_1 m_2 \frac{1}{4\pi r}$$

Z porównania do równań Einsteina dostaliśmy wcześniej $\lambda = \sqrt{8\pi G}$.
Ostatecznie otrzymujemy

$$E_{int} = -\frac{G m_1 m_2}{r}$$

Efekt Tolmana-Ehrenfesta-Podolsky'ego

$$\mathcal{H}_{int} = \lambda h^{\mu\nu} T_{\mu\nu}$$

gdzie

$$\begin{aligned} T_{\mu\nu} &= -\left(F_{\mu\lambda} F_{\nu}{}^{\lambda} - \frac{1}{4} \eta_{\mu\nu} F_{\rho\lambda} F^{\rho\lambda} \right) \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \end{aligned}$$

Inaczej można zapisać

$$\mathcal{H}_{int} = -\lambda h_{\mu\nu} \partial_\alpha A_\beta \partial_\gamma A_\delta K^{\mu\nu\alpha\beta\gamma\delta}$$

gdzie

$$K^{\mu\nu\alpha\beta\gamma\delta} = 2\eta^{\nu[\gamma}\eta^{\delta]\beta}\eta^{\mu\alpha} + 2\eta^{\nu[\delta}\eta^{\gamma]\alpha}\eta^{\mu\beta} + \eta^{\mu\nu}\eta^{\alpha[\delta}\eta^{\gamma]\beta}$$

$$
\begin{aligned}
S &= \mathcal{T} e^{-i\int d^4x \mathcal{H}_{int}} \\
&= 1 + \frac{(-i)}{1!}\int d^4x_1 \mathcal{H}_{int}(x_1) + \frac{(-i)^2}{2!}\int d^4x_1 d^4x_2 \mathcal{T}(\mathcal{H}_{int}(x_1)\mathcal{H}_{int}(x_2)) + \dots
\end{aligned}
$$

$$
\begin{aligned}
|in\rangle &= a^{\dagger}_{\lambda_1}(k_1)a^{\dagger}_{\lambda_2}(k_2)|0\rangle \\
\langle out| &= \langle 0|a_{\kappa_1}(p_1)a_{\kappa_2}(p_2)
\end{aligned}
$$

Wyraz $\mathcal{O}(\lambda^2)$:

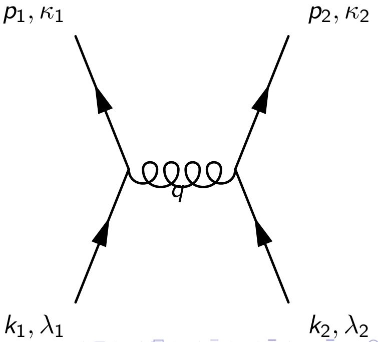
$$
\frac{(-i)^2}{2!}\langle 0|a_{\kappa_1}(p_1)a_{\kappa_2}(p_2)\int d^4x_1 d^4x_2 \mathcal{T}(\mathcal{H}_{int}(x_1)\mathcal{H}_{int}(x_2))a^{\dagger}_{\lambda_1}(k_1)a^{\dagger}_{\lambda_2}(k_2)|0\rangle
$$

$$\mathcal{T}(\mathcal{H}_{int}(x_1)\mathcal{H}_{int}(x_2)) =$$
$$\lambda^2 K^{\mu\nu\alpha\beta\gamma\delta} K^{\mu'\nu'\alpha'\beta'\gamma'\delta'} \mathcal{T}(h_{\mu\nu}(x_1)\partial_\alpha A_\beta(x_1)\partial_\gamma A_\delta(x_2) h_{\mu'\nu'}(x_2)\partial_{\alpha'} A_{\beta'}(x_2)\partial_{\gamma'} A_{\delta'}(x_2))$$

Nas interesuje człon

$$\overline{h_{\mu\nu}(x_1)\partial_\alpha A_\beta(x_1)\partial_\gamma A_\delta(x_2) h}_{\mu'\nu'}(x_2)\partial_{\alpha'} A_{\beta'}(x_2)\partial_{\gamma'} A_{\delta'}(x_2)$$

$$A = A(p_1, p_2) + A(p_2, p_1)$$



Wstawiamy

$$\overbrace{h_{\mu\nu}(x_1) h}_{\mu'\nu'}(x_2) = i \int \frac{d^4q}{(2\pi)^4} D_{\mu\nu;\mu'\nu'}(q) e^{-iq(x_1-x_2)}$$

i

$$A^{\mu}(x) = \int \frac{d^3k}{\sqrt{2(2\pi)^3\omega_k}} \sum_{\lambda=1}^{2} \left[a_{\lambda}(k)\epsilon^{\mu}(\lambda,k)e^{-ikx} + a_{\lambda}^{\dagger}(k)\epsilon^{\mu}(\lambda,k)e^{ikx} \right]$$

Całkując po pędach dostajemy

$$\begin{aligned} s_{if} &= -\frac{\lambda^2}{2} \int \frac{d^4x_1}{(2\pi)^3} \frac{d^4x_1}{(2\pi)^3} \frac{d^4q}{(2\pi)^4} \frac{K^{\mu\nu\alpha\beta\gamma\delta} K^{\mu'\nu'\alpha'\beta'\gamma'\delta'}}{\sqrt{2\omega_{k_1} 2\omega_{k_2} 2\omega_{p_1} 2\omega_{p_2}}} iD_{\mu\nu;\mu'\nu'}(q) \\ &\cdot \quad p_{\alpha}^1 \epsilon_{\beta}(\kappa_1, p_1) k_{\gamma}^1 \epsilon_{\delta}(\lambda_1, k_1) e^{ix_1(p_1-k_1-q)} \\ &\cdot \quad p_{\alpha'}^2 \epsilon_{\beta'}(\kappa_2, p_2) k_{\gamma'}^2 \epsilon_{\delta'}(\lambda_2, k_2) e^{ix_1(p_2-k_2+q)} \end{aligned}$$

$$
\begin{aligned}
s_{if} &= -\frac{\lambda^2 i}{16\pi^2}\frac{K^{\mu\nu\alpha\beta\gamma\delta}K^{\mu'\nu'\alpha'\beta'\gamma'\delta'}}{\sqrt{2\omega_{k_1}2\omega_{k_2}2\omega_{p_1}2\omega_{p_2}}}\frac{\eta_{\mu\mu'}\eta_{\nu\nu'}+\eta_{\mu\nu'}\eta_{\nu\mu'}-\eta_{\mu\nu}\eta_{\mu'\nu'}}{(p_1-k_1)^2} \\
\cdot &\quad p^1_\alpha\epsilon_\beta(\kappa_1,p_1)k^1_\gamma\epsilon_\delta(\lambda_1,k_1) \\
\cdot &\quad p^2_{\alpha'}\epsilon_{\beta'}(\kappa_2,p_2)k^2_{\gamma'}\epsilon_{\delta'}(\lambda_2,k_2) \\
\cdot &\quad \delta(p_1+p_2-k_1-k_2)
\end{aligned}
$$

Ogólnie mamy tu 70 członów.

Wybierzmy przypadek kiedy dwa fotony poruszają się równolegle w tym samym kierunku.

Czyli $k_1 \cdot k_2 = 0$

Likwidujemy w ten sposób 16 członów

Przejdźmy z $q \longrightarrow 0$, czyli fotony poruszają się daleko od siebie. Teraz człony z $k_1 \cdot p_1$, $k_1 \cdot p_2$, $k_2 \cdot p_1$, $k_2 \cdot p_2$ znikają .

Zostaje jeszcze 8 członow typu

$$k_1 \cdot \epsilon(k_2) \; k_1 \cdot \epsilon(p_2) \; k_2 \cdot \epsilon(p_1) \; p_1 \cdot \epsilon(k_1)$$

ale one też znikają bo $p_1 \longrightarrow k_1$.

Zostało 0 członów.

Fotony poruszające się w tym samym kierunku nie oddziaływują grawitacyjnie!

$$
\begin{aligned}
g^{\mu\nu} &= (\eta_{\mu\nu} + 2\lambda h_{\mu\nu})^{-1} = \\
&= \eta^{\mu\nu} - 2\lambda h^{\mu\nu} + 4\lambda^2 h^{\mu}_{\beta} h^{\beta\nu} - 8\lambda^3 h^{\mu\beta} h_{\beta\xi} h^{\xi\nu} + \dots
\end{aligned}
$$

Pamiętamy że $g_{\mu\nu} = \eta_{\mu\nu}(\delta^{\beta}_{\nu} + 2\lambda h^{\beta}_{\nu})$ i $\mathrm{det}A = e^{\mathrm{tr}A}$

$$
\begin{aligned}
\sqrt{-\mathrm{det} g} &= \sqrt{-\mathrm{det}\eta}\exp\left[\frac{1}{2}\mathrm{tr}\left(\delta^{\beta}_{\nu} + 2\lambda h^{\beta}_{\nu}\right)\right] \\
&= \exp\left[\frac{1}{2}\mathrm{tr}\left(2\lambda h^{\mu}_{\nu} - \frac{1}{2}(2\lambda)^2 h^{\mu}_{\tau} h^{\tau}_{\nu} + \frac{1}{3}(2\lambda)^3 h^{\mu}_{\tau} h^{\tau}_{\sigma} h^{\sigma}_{\nu}\right)\right] \\
&= \exp\left[\frac{1}{2}\left(2\lambda h^{\nu}_{\nu} - \frac{1}{2}(2\lambda)^2 h^{\nu}_{\tau} h^{\tau}_{\nu} + \frac{1}{3}(2\lambda)^3 h^{\nu}_{\tau} h^{\tau}_{\sigma} h^{\sigma}_{\nu}\right)\right] \\
&= 1 + \lambda h - \lambda^2 (h^{\mu}_{\nu} \bar{h}^{\nu}_{\mu}) + \mathcal{O}(\lambda^3)
\end{aligned}
$$

$$S = -\frac{1}{2\lambda^2}\int d^4x\sqrt{-g}(R-2\Lambda)$$

Z obserwacji mamy $\Lambda(G\hbar/c^3) \simeq 10^{-123}$.
Rozwijamy Lagranżjan

$$\mathcal{L} = \frac{\Lambda}{\lambda^2}\left(1+\lambda h - \lambda^2(h^\mu_\nu \bar{h}^\nu_\mu)\right) - \frac{1}{2\lambda^2}\Big(-\lambda^2\partial_\nu h_{\alpha\beta}\partial^\nu h^{\alpha\beta} + \lambda^2\partial_\nu h\partial^\nu h + \\ + -2\lambda^2\partial_\mu h\partial_\nu h^{\mu\nu} + 2\lambda^2\partial_\alpha h_{\nu\beta}\partial^\nu h^{\alpha\beta}\Big)$$

Dodając człon $(\partial^\mu h_\mu - \frac{1}{2}\partial_\nu h)^2$ dostajemy

$$\mathcal{L} = -\frac{1}{2}h_{\mu\nu}\Box h^{\mu\nu} + \frac{1}{4}h\Box h - \Lambda h^\mu_\nu \bar{h}^\nu_\mu + \frac{\Lambda h}{\lambda} + \frac{\Lambda}{\lambda^2}$$

Wprowadzając oznaczenie

$$V_{\mu\nu;\lambda\sigma} = -\frac{1}{2}(\eta_{\mu\lambda}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\lambda} - \eta_{\mu\nu}\eta_{\lambda\sigma})$$

mamy

$$\begin{aligned} -\frac{1}{2}h_{\mu\nu}\Box h^{\mu\nu} + \frac{1}{4}h\Box h &= \frac{1}{2}h^{\mu\nu}V_{\mu\nu;\lambda\sigma}\Box h^{\lambda\sigma} \\ -\Lambda h^{\mu}_{\nu}\bar{h}^{\nu}_{\mu} &= -\Lambda h^{\mu}_{\nu}(h^{\nu}_{\mu} - \frac{1}{2}\eta^{\nu}_{\mu}h) = \frac{\Lambda}{2}h^{\mu\nu}V_{\mu\nu;\lambda\sigma}h^{\lambda\sigma} \end{aligned}$$

Stąd

$$\mathcal{L} = \frac{1}{2}h^{\mu\nu}V_{\mu\nu;\lambda\sigma}(\Box + \Lambda)h^{\lambda\sigma} + \frac{\Lambda h}{\lambda} + \frac{\Lambda}{\lambda^2}$$

pojawia się człon masowy grawitonu, $m = \sqrt{\Lambda}$.

$$V \propto \frac{e^{-\sqrt{\Lambda}r}}{r}$$

Z obserwacji mamy

$$\Lambda(G\hbar/c^3) \simeq 10^{-123}$$

równoważnie

$$\sqrt{\Lambda} \simeq 10^{-28} cm^{-1}$$

wyrażając to w parsekach

$$1Mpc = 31 \cdot 10^{23} cm$$

dostajemy

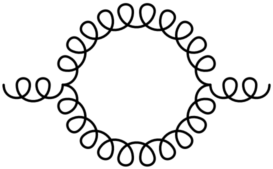
$$\sqrt{\Lambda} \simeq 3 \cdot 10^{-4} \ \ Mpc^{-1}$$

Rozbieżności

$$S = \int d^4x \left[\partial h \partial h + \lambda h \partial h \partial h + \lambda^2 h^2 \partial h \partial h + \ldots\right]$$

Pamiętamy że $\lambda = \sqrt{8\pi G} \sim \frac{1}{m_{pl}}$

$$\lambda^2\, h(x_1)\partial h(x_1)\partial h(x_1) h(x_2)\partial h(x_2)\partial h(x_2) \propto \lambda^2 \int^{\Lambda} d^4k \frac{k \cdot k \cdot k \cdot k}{k^2 \cdot k^2} \sim G\Lambda^4$$



$$\mathcal{M} \sim 1 + G\Lambda^2 + (G\Lambda^2)^2 + \ldots$$

$$\Lambda_{cr} \sim \frac{1}{\sqrt{G}} = m_{pl}$$

Nierenormalizowalność teorii grawitacji mówi nam że przy skali m_{pl} musi pojawić się nowa fizyka.